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A review on condition-based maintenance optimization models for stochastically deteriorating system

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ABSTRACT

Condition-based maintenance (CBM) is a maintenance strategy that collects and assesses real-time information, and recommends maintenance decisions based on the current condition of the system. In recent decades, research on CBM has been rapidly growing due to the rapid development of computer-based monitoring technologies. Research studies have proven that CBM, if planned properly, can be effective in improving equipment reliability at reduced costs. This paper presents a review of CBM literature with emphasis on mathematical modeling and optimization approaches. We focus this review on important aspects of the CBM, such as optimization criteria, inspection frequency, maintenance degree, solution methodology, etc. Since the modeling choice for the stochastic deterioration process greatly influences CBM strategy decisions, this review classifies the literature on CBM models based on the underlying deterioration processes, namely discrete- and continuous-state deterioration, and proportional hazard model. CBM models for multi-unit systems are also reviewed in this paper. This paper provides useful references for CBM management professionals and researchers working on CBM modeling and optimization.

1. Introduction

Industrial organizations are continuously seeking new strategies to improve the effectiveness of their operations. Maintenance optimization and the selection of maintenance strategies play an important role in the effectiveness of any industrial system's operation. Maintenance actions can be generally classified into two categories: corrective maintenance and preventive maintenance (PM). Traditionally, PM takes the form of system overhaul or unit replacement based on elapsed time, which is often referred to as time-based maintenance (TBM). TBM schedules are typically determined based on a probabilistic model of system failure. In recent years, condition-based maintenance (CBM) has received much attention in the maintenance research community. Unlike TBM policies that are developed based on historical failure data, CBM is a maintenance approach that emphasizes on combining data-driven reliability models with sensor data collected from monitored operating systems to develop strategies for condition monitoring and maintenance. The goal of CBM is to reduce unnecessary maintenance actions and eliminate the risks associated with preventive maintenance actions. Rapid development of computer based monitoring technologies (e.g., advanced sensors) has further facilitated CBM practices.

The literature on the use of mathematical modeling for the purpose of analyzing, planning, and optimizing TBM is abundant. Reviews on TBM can be found in [1–7]. In contrast, CBM has only received increasing attention recently, and only a few survey papers have considered CBM models extensively. The majority of existing CBM survey papers limits the scope within the diagnostic and prognostic methods and algorithms. For example, Jardine et al. [8] review recent studies and developments in CBM with emphasis on models, algorithms, and technologies for data acquisition and data processing. Peng et al. [9] divide the prognostic models into four categories: physical model, knowledge-based model, data-driven model, and combination model, and review various techniques and algorithms by this category. Ahmad and Kamaruddin [10] present an overview of time-based and condition-based maintenance in industrial applications, and summarize the most recent condition monitoring techniques. Shin and Jun [11] review CBM approach and address several aspects of CBM, such as definition, advantages and disadvantages, related international standards, procedures and techniques. Despite the recent rapid development of sensor technology that

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Nomenclature

CBM	Condition-based maintenance
CBPM	Condition-based predictive maintenance
DTMC	Discrete-time Markov chain
HMM	Hidden Markov model
IG	Inverse Gaussian
PHM	Proportional hazard model
PM	Preventive maintenance
POMDP	Partially observed Markov decision process
SMDP	Semi-Markov decision process
SPC	Statistical process control
TBM	Time-based maintenance

facilitates CBM, there exists increasing pressure on reducing unnecessary inspection and/or PM actions and the associated costs incurred from additional data collection, documentation, and analysis through optimal design of CBM policies. Therefore, mathematical modeling and optimization of CBM has become a major concern to operations and maintenance managers, and a review in this particular area is now more relevant. This paper aims at providing a review on CBM literature with emphasis on mathematical modeling approaches, and providing useful references for CBM management professionals and researchers working on CBM planning.

The popularity of CBM in the research community and industrial applications relies heavily on the development of stochastic deterioration models. We refer readers to Si et al. [12] and Ye and Xie [13] for reviews on degradation-based reliability models. The choice of the stochastic process that best describes the deterioration greatly influences the decision of the CBM strategy. In the literature, when the system condition is directly observable, stochastic deterioration models are usually classified based on whether deterioration states are discrete or continuous. For some systems that operate in dynamic environments, deterioration is caused by multiple factors referred to as covariates. For such systems, proportional hazard model (PHM) is commonly used to model the multivariate failure models. In this paper, we classify CBM models based on the three aforementioned classes: discrete, continuous, and PHM (see Fig. 1). Under this classification, we review existing CBM models for both single- and multi-unit systems.

The remainder of the paper is organized as follows. Section 2 reviews the CBM optimization models based on inspection quality, inspection frequency, optimization criteria, maintenance degree, and optimal design. Sections 3 and 4 review the mathematical modeling of CBM for single- and multi-component deteriorating systems, respectively. Finally, Section 5 concludes the paper with some future CBM research directions.

2. Condition-based maintenance modeling

The objective of CBM is typically to determine a maintenance policy that optimizes system performance according to certain criteria (i.e. cost, availability, reliability, etc.). The design of CBM policies has mainly focused on (1) inspection schedule and (2) preventive maintenance threshold. This section reviews the CBM optimization models based on inspection quality, inspection frequency, optimization criteria, maintenance degree, and optimal design.

2.1. Inspection quality

In practice, the majority of CBM models assume perfect inspections, i.e. each inspection reveals the exact state of the system without error. However, it is more realistic to assume that inspections are imperfect and may not detect failure states. Examples of studies that consider imperfect inspection can be found in CBM literature. Badıa et al. [14] propose an inspection policy to detect failures of a single-unit system subject to several hidden causes of failure, and assume that the probability of non-detection of a failure during an inspection depends on the failure cause. Lam [15] presents a CBM model for a deteriorating system in which the states of the system can be only diagnosed by non-perfect inspections. That is, an inspection is associated with probability of detection and probability of false alarm. Zequeira and Bérenguer [16] extend the previous models by classifying inspections into three types of inspections: perfect (detect without error all system failures), partial (detect without error only type I failures), and imperfect inspections (detect with error only type II failures) which may give false-positive results for other failure types. He et al. [17] present a special case of the model presented in [16] with no partial inspections are allowed. Berrade et al. [18,19] consider a similar approach in which inspection process is subject to error, and false positives and false negatives are possible.

Modeling imperfect inspection based on such inspection error classification is usually arbitrary because inspection error

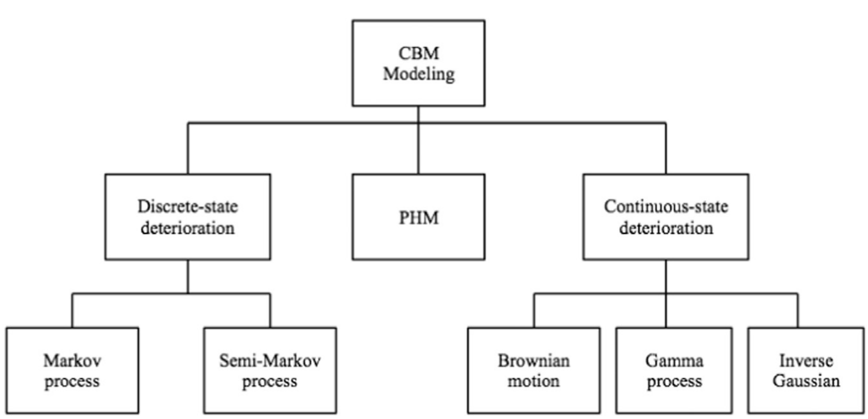

Fig. 1. CBM stochastic deterioration models.

probabilities are assumed constant while in reality they depend on deterioration process parameters. A more realistic modeling scheme is to consider inspection errors that depend on the parameters of the deterioration process. However, due to mathematical complexity, CBM models that consider imperfect inspection models based on system deterioration are very limited. van Noortwijk [20] reviews some imperfect inspection models based on Gamma degradation process where measurement errors are assumed to be normally distributed. Recently, a similar model for condition monitoring under imperfect inspection is presented by Ye et al. [21]. Similarly, Tang et al. [22] propose a RUL prediction method, which is central to CBM, based on a Wiener process with measurement error.

2.2. Inspection frequency

There are three main types of inspection schedules in CBM: continuous monitoring, periodic, and non-periodic inspection. In continuous monitoring, a continuous alarm system constantly monitors the condition of the machine and triggers a warning whenever something wrong is detected. Several studies develop CBM models for continuously monitored degrading systems – see examples in [23–27]. The advantage of continuous monitoring lies in the possibility of preventively maintaining the system only when necessary, thus, eliminating wasted inspection and maintenance activities. Continuous monitoring is often implemented in systems such as nuclear power plants, offshore installations, and aerospace components working under stressful conditions, and are typically continuously monitored because of the safety implications [28]. Besnard and Bertling [29] propose an approach to optimize CBM strategies based on inspection frequency for wind turbines, and show that online monitoring is the optimal strategy for high failure rate systems. Tian et al. [30] also propose a CBM policy for wind power generation systems in which wind turbine components are monitored continuously.

While continuous monitoring provides real-time information about the system states, it is always associated with high inspection costs, and a large volume of noise created by the continuous flow of data which might lead to inaccurate diagnostic [8]. There are also some systems where continuous monitoring is not applicable. For example, pipelines buried underground in oil and gas industries cannot be continuously monitored. Another example is reformer tubes in an ammonia plant which creep-rupture when their outside diameter enlarges beyond a certain threshold, however, their diameter can be only measured during annual shutdowns. For such systems, only periodic inspections are available or can be afforded. Optimal continuous-wear limit replacement under periodic inspections has been extensively studied – see examples in [31,32].

The choice of the inter-inspection times obviously influences the performance of the maintenance policy, e.g. cost and availability. In some industries, it might not always be worthwhile to inspect the system periodically, especially if the inspection procedure is costly (see Table 1). Therefore, several CBM studies have considered irregular inspection policies in their models. The following maintenance decision frame is usually adopted in these models. Upon each inspection, if the system fails, a corrective replacement is performed; if the system's cumulative deterioration is above a pre-determined maintenance threshold but below the failure threshold, a preventive replacement is performed; otherwise, the system is left unchanged. In all these cases, the next inspection interval is chosen based on the post-maintenance state of the system. The inspection interval decreases as the system deteriorates, which means more inspections should be carried out if the system is in a poor state. Examples on such non-periodic CBM policies can be found in [33,34].

A non-periodic inspection policy can lead to potential cost savings since inspections is performed less frequently in the early life of the system operation, and more frequently as the system ages. The disadvantage of non-periodic inspection schedules is that more documentation and rescheduling work is needed, and the risk of human errors significantly increases with additional rescheduling. A comparison among the three inspection policies is provided in Table 1.

2.3. Optimization criteria

There are several main performance measures widely used for CBM. The following subsections provide a review on CBM models based on the optimization criteria, including cost minimization, reliability or availability maximization, and multi-objective.

2.3.1. Cost minimization

Several cost functions have been proposed to evaluate CBM. In this section we discuss a few recent models involving cost functions for CBM programs. In these works, a maintenance cost model is proposed to find an optimal strategy leading to the minimum maintenance cost. Cost parameters in a cost model often include preventive replacement cost (C_p), corrective replacement cost (C_c), unit downtime cost (C_d), and inspection cost (C_i). Inter-inspection interval and preventive replacement threshold are two decision variables that influence the maintenance cost. Grall et al. [38] propose a CBM model that aims at finding optimal inspection times and replacement threshold in order to minimize the system total maintenance cost over an infinite horizon. Upon inspection, two decisions need to be made: (1) determine whether the system should be replaced preventively or correctively or left as is; (2) determine the time to the next inspection. The objective cost function is to minimize the long run *s*-expected cost rate EC_∞. Several models adapt the same maintenance decision rule aiming to minimize average long run maintenance cost, e.g. Fouladirad et al. [39] propose the same maintenance policy for systems subject to different modes of deterioration, and Fouladirad and Grall [40,41] for systems with dynamic deterioration rates. Let $N_i(t)$, $N_p(t)$, and $N_c(t)$ denote random number of inspections, preventive repairs, and corrective repairs in $[0,t]$, respectively. Let $d(t)$ represents time passed in a failed state in $[0,t]$. The cumulative maintenance cost is then given by [38]:

$$C(t) \equiv C_i N_i(t) + C_p N_p(t) + C_c N_c(t) + C_d d(t), \tag{1}$$

Table 1
Advantages and disadvantages of CBM inspection policies.

Inspection frequency	Advantages	Disadvantages	References
Continuous monitoring	Real time information on systems' state/ health conditions	High inspection costs; Unnecessary maintenance actions caused by inaccurate diagnostic	Liu et al. [28], and Jardine et al. [8]
Periodic inspection	Cost effective	May result in higher failure costs; More documentation work and difficult to implement	Tang et al. [35], and Huynh et al. [31]
Non-periodic inspection	Most cost efficient	May result in higher failure costs; More documentation work and difficult to implement	Flage et al. [36], and Lam and Banjevic [37]

and EC_∞ is given by

$$EC_\infty = \lim_{t\to\infty}\left[\frac{E\left[C(t)\right]}{t}\right] = C_i\lim_{t\to\infty}\left[\frac{E\left[N_i(t)\right]}{t}\right] + C_p\lim_{t\to\infty}\left[\frac{E\left[N_p(t)\right]}{t}\right] + C_c\lim_{t\to\infty}\left[\frac{E\left[N_c(t)\right]}{t}\right] + C_d\lim_{t\to\infty}\left[\frac{E\left[d(t)\right]}{t}\right]. \tag{2}$$

van der Weide et al. [42] propose a discounted cost model for optimizing CBM policies for engineering systems subject to random shock occurring randomly in time, and overall degradation is modeled as a cumulative stochastic point process. van der Weide and Pandey [43] extend the previous model by presenting a more generalized analysis of stochastic-shock damage process and apply this for the optimization of maintenance cost. Huynh et al. [31] develop a CBM model for a system that is subject to deterioration and traumatic shock events. They consider a periodic inspection/replacement policy, and find the expected cost rate for the inspection strategy.

2.3.2. Availability/reliability maximization

While cost is important in CBM, cost parameters are sometimes difficult to obtain. Time required for preventive/corrective maintenance and the uptime/downtime of the system can often be more accurately measured and easier to obtain. Therefore, availability is another practical performance measure for evaluating the effectiveness of a CBM policy.

Klutke and Yang [44] consider a periodic inspection policy for systems with non self-announcing failures. An expression for limiting average availability is derived by taking the ratio of the system expected up time to the expected time to first replacement, and opportunities for effective inspection strategies are suggested based on the availability model developed. Liao et al. [25] propose a condition-based availability limit policy to maximize the system's average short-run availability by monitoring its degrading state and setting a PM threshold. The expected uptime and downtime in a cycle, including preventive maintenance and replacement actions are:

$$E\left[\text{uptime}/(N \text{ maintenance in a cycle})\right] = E\left[T_1 + T_2 + \ldots + T_N + T_{N+1}\right], \tag{3}$$

and

$$E\left[\text{downtime}/(N \text{ maintenance in a cycle})\right] = E\left[M_1 + M_2 + \ldots + M_N + M_{N+1}\right] + \xi, \tag{4}$$

where T_i is the ith inter-maintenance time, M_i is the time required to perform maintenance i, ξ is the expected time required to perform a replacement, and T_{N+1} is the last operating duration before replacement. Therefore, the achieved availability is

$$\text{Availability} = \frac{E[\text{uptime}/(N \text{ maintenance in a cycle})]}{E[\text{downtime}/(N \text{ maintenance in a cycle})]}. \tag{5}$$

More examples on availability of continuously monitored systems can be found in [45–47].

2.3.3. Multi-objective

Cost, availability, reliability, or safety is commonly used as an optimization criterion in a CBM model. These different criterions are often used in insulation, and some of them may very well be in conflict. Hence, when faced with the challenge of simultaneously achieving multiple targets, some multi-criteria methods might be more appropriate and efficient. For example, for some multi-component systems, when the minimum system maintenance cost is obtained, the corresponding system reliability/availability may be so low that it may not be acceptable [48,49]. The reason behind this conflict is that, in practice, the various components in the system may have different maintenance costs and the importance of the reliability/availability of each in the system may be different. Therefore, both the system reliability/availability measures and maintenance cost must be considered together to obtain the optimal system maintenance policy.

In a multi-objective optimization problem, several possibly conflicting objective functions must be evaluated in correspondence of each decision variable. The goal is to identify the solution that gives the best compromise among the various objective functions. Ferreira et al. [32] develop a multi-criteria decision model where the expected cost of the inspection policy and the expected downtime are considered as criteria in the decision model. Zio and Viadana [50] consider the optimization of inspection intervals for components of a nuclear safety system. The optimization is sought with respect to different conflicting objectives functions: mean availability, cost of inspections/repairs, and exposure time of maintenance operators. Tian et al. [51] develop a multi-objective CBM optimization approach based on PHM, and use physical programming to deal with the multiple optimization objectives involved, which are system cost and reliability. The optimization approach systematically investigates the tradeoff between the optimization objectives, and provides the optimal solution that best represents the decision maker's preference. Van and Bérenguer [52] present a CBM policy for a deteriorating production system where degradation is assumed to be a Gamma process, and both maintenance cost and productivity are used in the optimization objective function.

Multi-objective optimization models of CBM have been also largely investigated for civil infrastructure systems, i.e. highway bridges. For example, Barone and Frangopol [53] propose a bi-objective optimal inspection and maintenance planning approach for structural systems to minimize both system failure rate and expected total cost. Barone and Frangopol [54] consider several multi-objective optimization problems for determining maintenance schedules for deteriorating structures. For each optimization problem, minimizing total cost is considered as the first objective, and the second objective include optimizing one of the following: reliability, availability, risk, and hazard function. Other examples of multi-objective models in this field include [54–58].

2.4. Maintenance degree

Most of the existing CBM models have been limited to perfect maintenance actions. However, PM activity is usually imperfect, for example, PM may bring the system back to a state between as-good-as-new and as-bad-as-old. Yet, only a limited CBM models have been developed with consideration of imperfect maintenance. These models can be classified into several classes. The first class is based on conducting minimal PM wherein the system is returned to its previous stage of deterioration. Hence, when the system is in deterioration stage k, minimal repair improves the system deterioration by one stage, and the system is returned to stage k–1, e.g. [59–62]. The second class is based on the “system degradation” in which the imperfect maintenance actions reduce the maintained system degradation by a random amount, e.g. [25,52,63–65]. Some of these models have considered the impact of imperfect maintenance actions on the system deterioration, including the residual damage, assuming that the deterioration level will be stochastically increasing as a function of the number of imperfect PM actions, e.g. [25,63]. Le and Tan [66] combine the system degradation approach with a probabilistic approach, where it is assumed that PM improves the state of the system to a better condition with a probability P. The third class is the improvement factor model which is based on the perception that maintenance actions will have different degree of impact on the system's deterioration rate, e.g. [27]. This approach is useful when the maintenance decision is in terms of the system hazard rate. However, for many models, the hazard rate function cannot be derived analytically, e.g. the non-stationary Wiener process. Alternatively, Zhang et al. [67] propose a random improvement factor model based on the assumption that maintenance actions will change the system rate of degradation rather than the level of degradation, and model the imperfect maintenance via the degradation rate function instead of the hazard rate function.

2.5. Optimal design

In CBM literature, decision variables often include inspection frequency and/or PM deterioration limits. Optimal design of inspection schedule has been studied in the literature, e.g. [32,37,68–70]. Some of these studies assume periodic inspection and seek the optimal inspection intervals, while others assume sequential inspection and find the optimal non-periodic inspection schedule. Optimal threshold design determination has been also considered, e.g. [25,47,71].

More often, optimal threshold design is determined together with the optimal design of inspection times. Tang et al. [35] propose a control limit policy where a preventive replacement threshold, the first inspection time, and the length of the subsequent regular inspection intervals are the decision variables in their maintenance policy. Zhao et al. [33] compare periodic and non-periodic inspection policies based on expected maintenance costs. Optimal threshold design together with the optimal design of inspection times have been studied also in the following studies [31,36,39–41,43,72,73].

There are also some studies that assume predetermined inspection times and consider the determination of the optimal maintenance action, which has to be made after inspection, i.e. replace, repair, or do nothing, e.g. [74,75]. Lam and Banjevic [37] propose a decision policy that provides an optimal non-periodic inspection schedule for CBM, and determines whether to replace the system or not at each decision point based on the current state of the system.

3. CBM for single-component systems

The literature on CBM has mainly considered single-component systems due to the complexity of the probabilistic analysis for multi-component systems. While some multi-component systems can be considered as single-component systems, as some components will fail more frequently than others, and not all components' failures will result in the failure of the other components or the whole system. There are also complex multi-component systems that contain multiple critical components, and it is important to develop appropriate CBM policies for such systems. In this section we review CBM models for single-component systems. Based on the underlying degradation models, we further classify CBM models into two categories: (1) CBM for discrete-state deterioration, and (2) CBM for continuous-state deterioration. Different deterioration models often lead to different modeling approaches and optimization techniques. CBM for a system with discrete-state deterioration is often formulated as a Markov decision process (MDP) or its variants. Standard algorithms used to solve MDP, such as value iteration algorithm or its variants, are usually adopted to seek the optimal maintenance policies, and structural properties of these optimal policies are analyzed accordingly. For systems with continuous-state deterioration, renewal theory or regenerative process is often used to model a CBM process. Non-linear optimization techniques, such as numerical search algorithms and heuristics, are commonly used to find the optimal or near-optimal CBM policies.

3.1. CBM for discrete-state deterioration

CBM models that assume discrete-state deterioration are usually modeled by Markov processes. Markov process is commonly used when precise measurements of the degradation states of the system cannot be obtained, and sometimes, it is a technical requirement since there is no need to work on every discrete value individually – from an engineering practice viewpoint. Instead, the degradation states are categorized into several deterioration levels. Semi-Markov process and the Hidden Markov process are used to handle more general reliability analysis [76]. Semi-Markov process is used when the problem can be solved by relaxing the strict conditions of Markov process. Hidden Markov process is used when the available information on the system is partially observed.

There exists abundant literature on systems under Markovian deterioration. However, relatively a few studies have considered the problem of prescribing optimal maintenance or replacement policies for such systems, especially, CBM policies. Kurt and Kharoufeh [70] consider the optimal replacement of a periodically inspected system under Markov deterioration that operates in a controlled environment. The deterioration status of the system is modeled as a discrete-time Markov chain (DTMC) on a finite state space $\Psi=(0, 1, \ldots, S)$, where the state space is structured in order of increasing deterioration levels. The transition probability matrix of this DTMC is governed by an environment process, which also evolves as a DTMC on a finite state space $\Upsilon=\{0, 1, \ldots, R\}$. The transition probability matrix of the system's deterioration status is denoted by $\mathrm{P}(r) = [p(s'|s, r)]_{s,s'\in\Psi}$, and the one-step transition probability matrix of the environment process is denoted by $\mathrm{Q} = [Q(r'|r)]_{r,r'\in\Upsilon}$. If inspection reveals that the system is failed, it must be replaced; otherwise, the system may either be replaced with a new one at a cost of $C(r, s)$, or left in operation for one more period with an immediate cost $\ell(r, s)$. Let $\vartheta(r, s)$ denote the minimum total expected discounted cost, given that the process starts in state $(r, s) \in \Upsilon \times S$, and $w(r, s)$ denote the total expected discounted cost of waiting in state $(r, s) \in \Upsilon \times S$. The optimal replacement problem is formulated as a discrete-time, infinite-horizon discounted Markov decision process (MDP). The optimality function is formulated as follows:

$$V(r, s) = \begin{cases} \min\{ w(r, s), V(0, 0) + C(r, s)\}, & \text{for } (r, s) \in \Upsilon \times S \\ V(0, 0) + C(r, s), & \text{for } r \in \Upsilon, s = S \end{cases}, \tag{6}$$

where for $(r, s)\in\Upsilon \times S$,

$$w(r, s) = \ell(r, s) + \lambda \sum_{r'\in\Upsilon} \sum_{s'\in S} P(s'|s, r)Q(r'|r)\left[\vartheta(r's, s') + I\right]. \tag{7}$$

Kurt and Kharoufeh [70] show that the optimal replacement policy exhibits control-limit characteristic with respect to the system's condition and its environment.

In practice, the physical condition of a system may not be known exactly, but can be estimated from the sensor signals from condition monitoring. Such estimations often do not reveal perfectly the system condition due to many reasons. For instance, measurement errors due to various faults, and/or noises in sensor signals can lead to inaccurate inspection results. A reasonable way to characterize the information with uncertainty is to specify a probability vector about the actual underlying condition. Byon and Ding [77] develop a model that incorporates the uncertain information from monitoring equipment using a partially observed Markov decision process (POMDP), and define the state of the system with probability distribution. Neves et al. [78] also consider the problem of constructing CBM policy for a system whose state is represented by a discrete-state Markov process and the condition measurement may not be perfect. Hidden Markov Model (HMM) theory is used for parameters estimation. Makis and Naderkhani [79] develop an optimal CBM policy for a partially observable system of which the degradation process is modeled as a continuous-time hidden Markov process, and formulate the problem in a POMDP framework.

3.1.1. Solution methodology

Markov decision process and Semi-Markov decision process (SMDP) are commonly used modeling approaches for CBM planning for discrete-state deterioration. Standard dynamic programming algorithms, i.e. value iteration algorithm, and its variants, such as policy iteration, modified policy iteration, etc., are widely used for solving these problems. Neves et al. [78] use value iteration algorithm to find an optimal repair rule that minimizes system operation cost. Makis and Naderkhani [79] formulate the CBM problem as a SMDP model and solve for an optimal maintenance policy using policy iteration algorithm. Tang et al. [35] present a maintenance policy for a deteriorating system subject to periodic inspection, where the optimization problem is formulated and solved in a SMDP framework using policy iteration algorithm.

3.2. CBM for continuous state deterioration

For some systems, it is easy to distinguish the different states, but the deterioration of many other systems are gradual over time, and it is difficult to classify the multiple states. Hence, modeling such systems as continuous-state systems is more realistic. CBM has been widely implemented for systems subject to continuous deterioration, since more condition/health information can be obtained through sensors that monitor the deterioration process. We mainly review CBM for commonly used stochastic deterioration models, such as Wiener process, Gamma process, and Inverse Gaussian process.

3.2.1. Wiener process

The Wiener process is appropriate in describing a degradation which shows increment/decrement of deterioration over time (non-monotonic), while Gamma process is more suitable in modeling monotonically increasing (or decreasing) degradation. Gorjian et al. [76] discusses the key merits, limitations, and applications of each degradation model.

The Wiener process $\{D(t),\ t\geq 0\}$ is a stochastic process with independent, real valued, normally distributed increments or decrements, and it is often expressed as [12]

$$D(t) = \mu t + \sigma W(t),\ \ \mu \geq 0, \tag{8}$$

where $W(t)$ is a standard Brownian motion, μ is the drift parameter, and σ is the diffusion coefficient of the process. The mean and the variance of $D(t)$ are given by μt and $\sigma^2 t$ respectively.

Guo et al. [63] propose a CBM policy for a mission-oriented system with fixed mission time τ based on a Wiener degradation process. The proposed model aims to determine the optimal PM threshold (l_a) to minimize the expected cost rate (C). The optimal PM policy is determined by the following optimization model:

$$\text{Min } C = \lim_{t\to\infty}\frac{E\big[\text{Total Cost}(t)\big]}{t}$$
$$\text{Subject to}\begin{cases} A1[j] \geq \xi, j = 1, \dots, k \\ T_{\text{op}(m)} \leq \text{U}_\tau, m = 1, \dots, n, \\ 0 < l_a < l_{\text{f}} \end{cases} \tag{9}$$

where $A1[i]$ is the average mission availability of the ith PM cycle, $T_{\text{op(m)}}$ is the expected operating time for renewal cycle m, ζ is the CM duration time, U is the maximum mission number before renewal, and l_f is the failure threshold. Other CBM models based on Wiener degradation process include [80,81].

3.2.2. Gamma process

For some systems, the degradation process is usually irreversible, i.e. when degradation is in the form of cumulative damage, Gamma process is an appropriate model. Gamma process has a monotonic degradation path, and has been studied extensively in CBM models for continuously deteriorating systems (see van Noortwijk [20] for a review and examples).

A basic Gamma process model $\{X(t),\ t\geq 0\}$ is a stochastic process with independent and Gamma-distributed increments such that $\Delta X(t){=}X(t{+}u){-}X(u)$ follows Gamma$(\upsilon(t{+}u){-}\upsilon(u),\ \mu)$ with probability density function [13]:

$$f_{x(t)}(x;\ \mu,\ v) = \frac{\mu(\mu y)^{v(t+u)-v(u)-1}}{\Gamma(v(t+u)-v(u))}\exp(-\mu y), \tag{10}$$

where μ, $\mu \geq 0$ is the scale parameter, and $\upsilon(t)$ is the shape function which is required to be a non-decreasing real valued function for $t\geq 0$.

Recently, Caballé et al. [82] propose a periodic-inspection/replacement strategy for a system subject to internal degradation and external sudden shocks. The internal degradation process follows a non-homogeneous Poisson process and its growth is modeled using a Gamma process. Do et al. [83] present a proactive CBM policy considering maintenance actions for a Gamma deterioration process. The proposed model aims to determine the optimal maintenance action (perfect and imperfect), and the inspection interval. They consider a maintenance policy in which the system will be preventively maintained when its deterioration level is between the PM threshold (M) and the failure threshold (L). It is assumed that only the Kth PM action is perfect, and all previous PM actions are imperfect. The cumulative maintenance cost at time t is a function of decision parameters K, M, and Q (failure probability between two inspection times),

$$C^t(K, M, Q) = C_i N_i(t) + \sum_{k=1}^{N_{ip}(t)} C_p^k + C_p N_p(t) + \sum_{j=1}^{N_c(t)} C_c + C_d d(t). \tag{11}$$

where $N_i(t)$, $N_p(t)$, $N_p(t)$, $N_c(t)$ are the number of inspections, PM actions, imperfect maintenance actions, and corrective replacements in [0,t], respectively, and C_i, C_p, C_p^k, C_d, are the inspection cost, perfect PM cost, kth imperfect maintenance cost, corrective replacement cost, and system downtime cost, respectively, and $d(t)$ is the total time passed in a failed state in [0,t].

Alternatively, some studies assume that system's deterioration follows an exponential process, as a special case of Gamma, to make it easier analytically. Castanier et al. [84] present a CBM policy for a system subject to continuous deterioration modeled by an exponential model, and obtain optimal thresholds and inspection schedule based on cost and availability criteria. Deloux et al. [85] present a joint statistical process control (SPC) and CBM policy. CBM inspects and replaces the system according to observed deterioration level, and SPC is used to monitor stress covariate.

3.2.3. Inverse Gaussian process

The inverse Gaussian (IG) process has gained significant attention recently. IG process was introduced by Wang and Xu [86], and further investigated by Ye and Chen [87]. The IG process is a limiting compound Poisson Process that is suitable for modeling heterogeneous degradation of systems deteriorating in a random environment, i.e. it is flexible in incorporating random effects and covariates that account for heterogeneities [87]. Similar to the

Gamma process, the IG process is also suitable in modeling monotonic degradation, but is more flexible in incorporating random effects compared to the Gamma process.

The IG process $Y(t)$, $t \geq 0$ is a stochastic process with independent and IG-distributed increments such that $Y(t) \sim \text{IG}(\mu\Lambda(t), \lambda\Lambda^2(t))$, where $\Lambda(t)$ is nonnegative and monotone increasing, and $\text{IG}(a,b)$, a, $b > 0$ is the IG distribution with probability density function [87]

$$f_{\mathcal{IG}}(y;\, a,\, b) = \sqrt{\frac{b}{2\pi y^3}} \exp\left[-\frac{b(y-a)^2}{2a^2 y} \right], \quad y > 0. \tag{12}$$

Despite the increasing attention that IG process has received recently [9,86–89], it is still new in degradation modeling, and there is scarce literature on CBM based on IG processes. Chen et al. [88] propose a periodic inspection/replacement model for heterogeneous degradation which conforms to an IG process with random effects, and find the optimal CBM policy. The policy aims to find optimal inspection interval and corresponding maintenance to minimize total operation costs.

3.2.4. Solution methodology

CBM problems for continuous-state deteriorations often use cost-based criteria. Such cost-based criteria are all based on renewals bringing a component or structure back to its original condition, hence, renewal (reward) theory is used to compute them [63]. A detailed review on CBM models for Gamma process deterioration using renewal theory can be found in van Noortwijk [20]. The CBM cost rate functions for stochastic degradation are often mathematically complex, and heuristics are typically used to overcome the difficulty of solving the optimization problem. Do et al. [83] use Monte Carlo simulation to find the optimal decision parameters: PM threshold (M), imperfect PM threshold (K), and failure probability between two inspection times (Q), that minimize the long run expected maintenance cost per unit of time.

Alternatively, some studies use continuous-state semi-regenerative (Markov renewal) techniques instead of the classical renewal theorem to simplify the analysis of system behavior, e.g. [38,41,84,90]. Note that if a system's continuous states can be aggregated into several distinct discrete states, modeling approaches for discrete-state deterioration can also be appropriate. For example, Chen et al. [88] aggregate continuous states governed by an IG process into a discrete state Markov chain and formulate the maintenance problem as MDP.

3.3. CBM for proportional hazard model

In some components that operate in dynamic environments, deterioration is caused by multiple factors called covariates (temperature, vibration, running speed, etc.). Since these covariates change stochastically and may influence the lifetime of the component, all covariates should be included in the modeling. One approach for modeling the lifetime of a component in a dynamic environment is to describe the component's failure rate by a stochastic process that is referred to as a hazard rate process [91]. Thus, dynamic multivariate failure models such as covariate-based hazard models were developed. PHM is a commonly used CBM covariate-based hazard model, which takes into account both equipment age and condition when calculating failure rate. A PHM with time-dependent covariates has a hazard function of the form [12]:

$$h(t, X(t)) = h_0(t)\exp(\gamma X(t)), \tag{13}$$

where $h_0(t)$ is a baseline hazard function, $X(t)$ is the vector of covariates at time t, and γ is the vector of coefficients.

PHM has been a popular model in CBM due to its suitability for analyzing both event and condition monitoring data together. Zhao et al. [33] propose a CBM model that is similar to the PHM model, and derive the inspection/replacement policy that minimizes the expected average maintenance cost. Tang et al. [92] present a CBM policy for degrading system subject to soft and hard failures that is monitored by periodic inspections. The hard failure is modeled by PHM, and the SMDP framework has been developed to find the optimal failure rate-based maintenance policy. Recently, Lam and Banjevic [37] propose a CBM policy for a system with covariates and use PHM to represent the risk of failure.

3.3.1. Partially observable systems

In PHM, the monitored systems are classified, based on monitoring quality, into two categories: completely observable systems where the system condition can be completely observed, and partially observable systems where the system condition cannot be fully observed. When there is uncertainty concerning the condition of the system, the degradation process is typically modeled by a POMDP or a HMM. Ghasemi et al. [74] formulate the CBM problem as POMDP and use the imperfect information obtained at inspection to calculate the probability of the system being in a certain state. They find the system's optimal replacement policy using dynamic programming. Ghasemi et al. [93] further use PHM to find the optimal CBM policy for a system under imperfect inspection. The system's degradation is modeled using HMM and dynamic programming is used to determine the optimal interval between inspections and the corresponding replacement criterion.

4. CBM for multi-component systems

As stated in Section 3, existing literature on CBM has mostly focused on single-component systems. Only a few CBM studies have developed maintenance policies for multi-component systems as the probabilistic analysis and the determination of optimal CBM policies for multi-component systems are much more difficult than those for single-component systems [94]. Wang (2002) reviews literature on maintenance policies for multi-component systems, with focus on PM rather than on CBM. In theory, a CBM policy for a single-component system maybe applicable to a multi-component system by selecting a one-dimensional “health index” of the system instead of its complete multi-dimensional state [38]. In this case, the maintenance decision is determined at system level. Such a decision is suitable to systems whose components are independent of each other. However, most of the existing CBM strategies are done at component level, that is, the optimal CBM policy for a single-component system is employed per component in the multi-component system, e.g. [30,47,95].

In practice, a multi-component system is usually subject to dependencies among components, i.e. economic, structural, and stochastic dependence. Therefore, CBM strategies for single-component systems cannot be properly applied to multi-component systems. Several CBM studies have been developed considering dependencies among components. Some of these studies, e.g. [30,84,95–101], study CBM strategies with economic dependence based on the fact that costs can be reduced when several components are jointly maintained. While other studies, e.g. [75,80,94,102–104], study strategies with stochastic dependence where the failure or the degradation of a component can affect the state of one or more other components. CBM studies on multi-component systems considering structural dependence are limited, as it could involve more difficult analytical formulation. However, ignoring system structure in the maintenance decision-making may not always guarantee the best maintenance performance. Furthermore, modeling the combination of dependencies between components is rare in CBM literature, since combining more than one makes the models too complicated to analyze and solve. Li et al. [105] propose a CBM policy for multi-component systems taking into account inter-component stochastic and economic dependencies. Van Horenbeek and Pintelon [106] incorporate structural dependence along with economic and stochastic dependencies into their CBM model. Xia et al. [107] propose a CBM policy for multi-unit series systems that decreases the frequency of failures and produces a cost-effective schedule. Jiang et al. [108] and Olde Keizer et al. [109] consider opportunistic CBM strategies for systems with economic dependencies and redundancy. Azadeh et al. [110] evaluate the effectiveness of CBM policy compared to corrective maintenance and PM policies in a series-parallel multi-component system, and conclude that if established properly, CBM policy is more effective in terms of cost and reliability. Finally, a very few studies have considered multi-level decision-making for multi-component systems. Huynh et al. [111] present a multi-level decision-making process that combines decisions at both system level and component level while considering economic and structural dependencies. Nguyen et al. [112] propose a similar multi-level decision approach taking into account economic dependencies, in which system level aims at triggering maintenance interventions, and component level addresses the optimal selection of a group of several components to be preventively maintained.

5. Conclusion and future directions

In this paper, we review recent CBM literature with emphasis on mathematical modeling and optimization approaches. In particular, we focus on optimal design of CBM policies including inspection frequency, inspection/maintenance quality, and optimization criterion. We also review CBM models based on their underlying deterioration models: discrete- and continuous-state degradation models. CBM models for PHM are also reviewed in this paper. We have focused our review and analysis mainly on single-unit systems, however, several CBM models for multi-unit systems are also included and reviewed to provide a more complete reference for researchers and professionals that are interested in CBM. This review provides a large number of references that can support development of better CBM models.

CBM has gained increasing attention recently as a preferred approach to PM, however, further research on CBM is still in great need. First, most of the existing CBM researches focus on single-component systems, and CBM for multi-component systems has not been adequately addressed. One potential direction is to develop CBM policies for multi-component systems with different components types, as opposed to identical, and consider the degradation dependencies among the components. It will also be interesting to incorporate the health state of the global system into CBM decision-making, by considering a multi-level decision-making process that combines CBM decisions at both system and component level. Second, in practice, many systems are subject to multiple degradation processes, which can be internal, external, dependent or independent. Multiple degradation processes impose great challenges on CBM due to mathematical complexity. Therefore, models considering multiple degradation processes are scarce in CBM literature, and the only few existing models mainly consider multiple independent degradation processes, e.g. [82,113]. There is still a need to study CBM in systems with multiple failure causes from different degradation sources, and analyze/model dependency between the degradation processes. Third, human errors are normally involved in different functions of CBM, including condition monitoring and maintenance, which can degrade the effectiveness of CBM strategies. Only a few CBM studies have investigated human error in CBM modeling [114]. This creates a need for models that integrate human reliability into CBM optimization models to investigate the effectiveness of CBM strategies in presence of human interference. Lastly, Prognostic and Health Management is another critical facet of CBM that is needed to move toward advanced technologies application in health monitoring of complex and sensitive equipment, e.g. heavy vehicles, pumps, and wind turbines [30,96]. This consists of technologies and methods that assess system reliability based on its actual life cycle conditions to predict incipient failure and mitigate system risk [115]. With the development of Prognostics and Health Management techniques, a new trend of maintenance strategies called condition-based predictive maintenance (CBPM) has recently emerged [11,116]. CBPM utilizes future machine conditions predictions (prognoses) instead of diagnostic information, and make maintenance decisions accordingly [27,117]. It is worthwhile to develop new models and methodologies that adaptively determine inspection schedule and maintenance threshold(s) based on predicted failure times through Prognostics and Health Management.

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