Evaluating military training aircrafts through the combination of multi-criteria decision making processes with fuzzy logic. A case study in the Spanish Air Force Academy

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A B S T R A C T

The combination of Multi-Criteria Decision Making (MCDM) processes with Fuzzy Logic can be efficiently applied to solve decision problems with criteria differing in nature. This combination is used in the present work to solve a real world decision problem of interest for the Spanish Air Force, specifically, the selection of the best military training aircraft based on a set of criteria. This decision problem involves, on one hand, quantitative or technical criteria (service ceiling, endurance, etc.) and, on the other hand, qualitative criteria (human factors, flying and handling qualities, etc.) based on the experience of a set of senior pilots and flight instructors of the Spanish Air Force collected via surveys. In order to extract information from the expert surveys, the MCDM process was combined with fuzzy logic. The Analytic Hierarchy Process (AHP) was used to obtain the weights of the criteria and, through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), the alternatives were evaluated. This work is a preliminary study of the training aircraft alternatives proposed by the Air Staff and the Logistic Support Command of the Spanish Air Force. These alternatives were chosen based on operational criteria which are detailed in the work. As a result of the decision process used, the best alternative was shown to be the Pilatus PC-21 aircraft.

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1. Introduction

When an Air Force needs to select a training aircraft, decision-making is a crucial intellectual activity involved in the process, as in some stages of aircraft design [34]. Generally, in the decision-making process for these problems, a large number of key and different criteria are involved. Therefore, it is advisable to use tools for their resolution such as Multi-Criteria Decision Making (MCDM) processes, whose use is widespread nowadays in the military [23] and aerospace fields [45], as well as in other research disciplines [18,41].

In addition, the criteria for the optimal choice of a training aircraft exhibit different nature, including quantitative criteria (service ceiling, stalling speed, endurance, etc.) as well as qualitative criteria (human factors, flying and handling qualities, etc.). Thus, in order to solve this optimization problem, the application of any of the known MCDM methods as the Analytic Hierarchy Process (AHP) [40], the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [22], the ELImination Et Choix Traduisant la REalité (ELECTRE) method [39], etc., must be combined with fuzzy logic [19,21,27,50] with the aim of being able to perform the extraction of knowledge and solving the proposed problem.

Several examples of the application of the MCDM in the military field can be found in the scientific literature. In the 90s, some weapon systems were evaluated using ranking fuzzy numbers with AHP [8], a methodology also exploited by the United States Army to manage organization in peacekeeping force design in Bosnia [35]. Nowadays, this methodology has proven to be a useful tool, e.g., for determining the size of the USA destroyer fleet in case of an armed conflict in the Korean peninsula during 2015 [12].

From the point of view of aircraft selection, the AHP methodology has been used to evaluate combat helicopters [9], as well as the quality in combat aircrafts maintenance tasks [48]. Regarding the TOPSIS method, it has been used to evaluate training aircrafts in China [46]. A combination of the two aforementioned methodologies has been used to determine the efficiency of combat aircrafts [47].

From the perspective of the Spanish Air Force (SPAF), the Air Staff (EMA) and the Logistics Support Command (MALOG) are the main decision-makers. These groups will adopt a decision based on several operative, politics, strategic, and economics viewpoints. Nevertheless, it is advisable to make a preliminary assessment...
taking into account both the most significant technical criteria and the experience of important advisory groups such as senior test pilots and flight instructors of the SPAF.

The aim of this work is the selection of the best military training aircraft among several alternatives found on the current aircraft market. This problem will be faced by using the AHP methodology to obtain the weights of the criteria that influence the decision, and the TOPSIS method to evaluate the different alternatives. Furthermore, these two methods will be combined with fuzzy logic due to the existence of qualitative and quantitative criteria.

The rest of this paper is structured as follows. In Section 2, the methodologies used for obtaining the weights of the different criteria and for carrying out the evaluation of alternatives are described. Section 3 presents a real-world training aircraft selection problem currently faced by the Spanish Air Force Academy (SPafa), and the optimum result obtained with the proposed methodology. Finally, Section 4 collects the conclusions reached, as well as possible future research lines.

2. Methodology

On countless occasions, human beings must select, among several alternatives, the one they think is the best option. A decision problem arises when, in response to a proposed situation, there are two or more alternatives from which, individually or collectively, it is necessary to choose one of them or, at least, sort the preferences. Typically, decision-making processes are based on the knowledge and the experience of the decision maker in similar situations which occurred before. Nevertheless, it is not common to use any methodology or tool to assist in the decision-making process, such as the valuable MCDM tools, whose use for military and aerospace problems has been introduced in Section 1.

2.1. The statement of a decision problem

Any MCDM can be expressed by means of five elements \( \{C, D, r, \prec, I\} \), where:

1. \( C = \{C_1, C_2, \ldots, C_m\} \) is the set of criteria that allow the comparison of the alternatives from specific points of view.
2. \( D = \{D_1, D_2, \ldots, D_n\} \) is the set of feasible alternatives for the decision-maker, and from which the decision-maker must choose one.
3. \( r : D \times C \rightarrow R \) is a function which for every decision \( D_i \) and every criterion \( C_j \) gives a value such that \((D_i, C_j) \rightarrow r(D_i, C_j) = r_{ij}\). Once the sets of criteria and alternatives have been chosen, a measure of the effect produced by each alternative with respect to each criterion is needed. By means of linguistic terms, the decision-maker represents the goodness of an alternative with respect to a criterion; different values of \( r \) can be represented by means of a matrix called the matrix of decision making.
4. \( \prec \) is the decision-maker’s relation of preferences. A coherent decision-maker is assumed, therefore he should try to maximize his profits or, at least, minimize his losses. In this case the decision-maker needs to obtain the best alternative according to the considered criteria.
5. \( I \) is certain information about the criteria, which in this study will be linguistic. The decision-maker gives linguistic information about the importance for each criterion.

Particularly, in this study, the sets \( C \) and \( D \) are finite, this fact will avoid convergence and measurability problems.

2.2. Linguistic variables and fuzzy sets

2.2.1. Linguistic variables

Most of the time, the decision-maker is not able to define the relative importance of the criteria or the goodness of the alternatives with respect to each criterion in a quantitative way. In these situations, approximate measures or quantities can be used as in [13,31]. Another way to overcome this problem is by applying the concept of fuzzy sets introduced by Zadeh [50]. Such fuzzy logic can be applied through linguistic variables [51].

A linguistic variable [52,53] refers to a variable whose values are words or sentences in a natural or artificial language. In general, the evaluation of judgments by means of linguistic terms is easier for the decision-maker than trying to quantify them. In these cases, using fuzzy numbers is more appropriate than using real numbers for evaluating the assessments.

2.2.2. Fuzzy sets

In this study, linguistic variables will be identified with fuzzy sets [5,24,26]. Fuzzy set theory was developed to deal with vague, imprecise, and uncertain problems [50]. It has been used as a modelling tool for complex systems that can be controlled by humans but are difficult to define precisely [26]. Within this formulation, a collection of objects (universe of discourse) \( X \) is related to a fuzzy set \( A \) described by a membership function \( f_A \) with values in the interval \([0,1]\).

\( f_A : X \rightarrow [0,1] \)

Thus \( A \) can be represented as \( A = \{f_A(x) \mid x \in X\} \). The degree with which \( x \) belongs to \( A \) is the membership function \( f_A(x) \). In this work, triangular membership functions, related to triangular fuzzy numbers \((a, b, c)\), will be used. The interested reader can find a detailed description of these numbers, as well as the arithmetic operations on them, in [27]. An approach for defuzzification after applying the MCDM can be found in [17].

2.3. Analytic Hierarchy Process (AHP)

This MCDM tool, developed by Saaty [40], is a pairwise comparison method that attempts to estimate the impact of each one of the alternatives in a set on the overall importance of a hierarchy of criteria.

For the defined set of criteria \( C = \{C_1, C_2, \ldots, C_m\} \), let’s denote their actual weights by \( w_1, w_2, \ldots, w_m \). The matrix of pairwise comparisons \( A = [a_{ij}] \) collects an expert’s preference between individual pairs of alternatives “i” and “j”, i.e., \( a_{ij} \) is the relative importance (in the opinion of the expert) of \( C_i \) to \( C_j \). Consequently, the elements \( a_{ij} \) can be considered as estimators of the ratios “\( w_i/w_j \)”. According to Saaty [40], the elements \( a_{ij} \in \{1/9, \ldots, 1, \ldots, 9\} \), are positive and satisfy the reciprocity property: \( a_{ij} = 1/a_{ji} \) (i, j = 1, 2, …, m). Obviously, the elements on the main diagonal are \( a_{ii} = 1 \).

In case of using linguistic variables and fuzzy numbers, as in this work, elements “\( a_{ij} \)” are fuzzy numbers. Table 1 presents the linguistic decision-maker’s preferences in the pair-wise comparison process used in this study.

Once the matrix \( A \) is obtained, the vector of weights is the eigenvector corresponding to the maximum eigenvalue \( \lambda_{max} \). This vector of weights allows the quantification of the importance of the different criteria.

Additionally, the maximum eigenvalue can be used as a measure of the consistency of the expert’s preferences arranged in the comparison matrix. The consistency index (CI) is given by \( CI = \lambda_{max} - m/(m-1) \). If the expert shows some minor inconsistency, then \( \lambda_{max} > m \). Additionally, Saaty [40] proposes the following measure of the consistency ratio: \( CR = CI/RI \), where RI (random
index) is the average value of CI obtained in [3] from random matrices using the Saaty scale in [15] as shown in Table 2. According to [40], the matrix is accepted as a consistent one if CR < 0.1.

In AHP problems with fuzzy numbers, the geometric normalized average is used as an estimator of the weights instead of the eigenvector related to \( \lambda_{\text{max}} \). This geometric normalized average is calculated by the following expression:

\[
W_i = \frac{\left( \prod_{j=1}^{m} (a_{ij}, b_{ij}, c_{ij}) \right)^{1/m}}{\sum_{j=1}^{m} \left( \prod_{j=1}^{m} (a_{ij}, b_{ij}, c_{ij}) \right)^{1/m}}
\]

where \((a_{ij}, b_{ij}, c_{ij})\) is a fuzzy number.

Finally, to obtain the vector of weights, the following normalizing operation is used:

\[
(W_{c_1}, W_{c_2}, W_{c_3}) = \left[ \frac{c_{i_1}}{\sum_{j=1}^{n} c_{i_j}}, \frac{c_{i_2}}{\sum_{j=1}^{n} c_{i_j}}, \frac{c_{i_3}}{\sum_{j=1}^{n} c_{i_j}} \right]
\]

2.4. The TOPSIS method

The TOPSIS method, proposed by Hwang and Yoon [22], is one of the best known classical MCDM methods. It is based upon the concept that given a set of alternatives, the one chosen should have the shortest distance from the positive ideal solution (PIS), and the longest distance from the negative ideal solution (NIS). This approach is exploited for four reasons [46]:

a) TOPSIS logic is rational and understandable;
b) The computational processes are straightforward;
c) The concept permits the pursuit of best alternatives for each criterion depicted in a simple mathematical formulation, and
d) The importance weights are incorporated into the comparison procedures.

In this study, the TOPSIS method is used to select the preference order of the alternatives. The existence of both numeric and linguistic labels (related to fuzzy numbers) must be taken into account. Thus, a fuzzy TOPSIS method must be used. The adaptation of the operations associated to fuzzy numbers to derive a fuzzy TOPSIS method from the generic TOPSIS method can be found in [16]. The computational steps of the fuzzy TOPSIS algorithm used in this work are summarized in Fig. 1.

3. A decision problem: assessment of military aircrafts

3.1. Spanish Air Force Academy needs

The main goal of the SPFA is to train the future SPAF officers by providing them with academic, military, and aeronautical training. Regarding the aeronautical training, the SPFA is responsible for the two first stages (elementary and basic training) in the training program for military Spanish pilots.

Nowadays, there are two training aircrafts at the SPFA: the ENAER T.35C Tamiz for the elementary course, and the CASA C-101 Aviojet for the basic course [1]. These aircrafts have been in service for more than 30 years, much longer than initially planned. Consequently, the C-101, a basic and advanced training jet, has so many flight hours that severe deficiencies have arisen during the last years. From the structural point of view, corrosion and fatigue problems have been detected, which have greatly increased the maintenance costs. Regarding its avionics, failures in both internal and external communications equipment are common, certain systems exhibit low reliability, and the communication and navigation systems are obsolete. Additionally, the maintenance costs of the training weapon systems have grown in recent years. As a result, the SPFA has detected the need to replace these training jets as soon as possible.

As in other modern Air Forces, the Spanish Air Staff (EMA) and the Logistic Support Command (MALOG) are completely aware of the fact that training efficiency strongly depends on the choice of a modern “state of technology” training aircraft. Nowadays, there is a tendency to use modern turbopropellers with a wide flight envelope, large power, performance and flight qualities close to jet aircrafts, and last generation avionics and security systems. Potential substitutes should also be able to replace the ENAER T.35C for the elementary training in the future. This would enable the use of the same training aircraft form the elementary phase to the begin-

### Table 1

<table>
<thead>
<tr>
<th>Labels</th>
<th>Verbal judgments of preferences between criterion i and criterion j</th>
<th>Triangular fuzzy scale and reciprocals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ci</td>
<td>Cj is equally important</td>
<td>(1, 1, 1)/(1, 1, 1)</td>
</tr>
<tr>
<td>(M + i)</td>
<td>Cj is slightly more/less important than Cj</td>
<td>(2, 3, 4)/(1, 4, 3, 1/2)</td>
</tr>
<tr>
<td>(M + i)</td>
<td>Cj is extremely more/less important than Cj</td>
<td>(4, 5, 6)/(1, 6, 1, 5, 1/4)</td>
</tr>
<tr>
<td>(Mu + i)</td>
<td>Cj is very strongly more/less important than Cj</td>
<td>(6, 7, 8)/(1, 8, 1, 7, 1/6)</td>
</tr>
<tr>
<td>(Ex + i)</td>
<td>Cj is extremely more/less important than Cj</td>
<td>(8, 9, 9)/(1, 9, 1, 9, 1/8)</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>RI</th>
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<tr>
<td>0.00</td>
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<tr>
<td>0.5247</td>
<td>2</td>
</tr>
<tr>
<td>0.8816</td>
<td>3</td>
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<tr>
<td>1.1086</td>
<td>4</td>
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<td>1.2479</td>
<td>5</td>
</tr>
<tr>
<td>1.3417</td>
<td>6</td>
</tr>
<tr>
<td>1.4057</td>
<td>7</td>
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<tr>
<td>1.5838</td>
<td>14</td>
</tr>
</tbody>
</table>

Fig. 1. The fuzzy TOPSIS algorithm.
ning of the advanced phases resulting in a reduction of flight hours in the advanced training course. Additionally, the huge technological gap between the trainer and the 4th generation of fighters (Eurofighter TYPHOON in the SPAF) would be greatly reduced.

3.2. Alternatives definition

In 2008, the SPAF EMA a basic training aircraft selection process taking into account SPAFA needs and requirements in order to determine the best candidate to replace the current CASA C-101 Aviojet. The proposed alternatives were:

- **Alternative A1. Pilatus PC-21.** This swiss-enterprise trainer exhibits good aerodynamic efficiency. Its power plant is the Pratt & Whitney PT6A-68B (1600 shp) engine. Its avionics system can address the current and future training demands. Other notable characteristics are: a pressurized cockpit, air conditioning, an anti-g system, and on-board oxygen generation [36]. Nowadays, this aircraft is in service in the Swiss Air Force and the Republic of Singapore Air Force, and, recently, the United Arab Emirates Air Force and Saudi Arabia have signed contracts to acquire 25 and 55 of these aircrafts, respectively [37].

- **Alternative A2. Beechcraft T-6C.** This aircraft can be used for basic and advanced training. It is motorized by the Pratt & Whitney PT6A-68 (1100 shp) turboprop. It includes an advanced Esterline CMC all-glass cockpit, a SparrowHawk Head-Up Display (HUD), an on-board oxygen-generating system, 0/0 ejection-seat survival kit, the integrated Up Front Control Panel (UFCP), and an embedded synthetic training system [44]. This trainer is widely used in many current worldwide military programs, such as NATO Flying, Training in Canada, the Israeli Air Force, the Hellenic Air Force of Greece, the Iraqi Air Force, the Mexican Air Force, the Royal Moroccan Air Force [4], and the Royal New Zealand Air Force [2].

- **Alternative A3. PZL-130 Orlik (TC-II).** This Polish aircraft [38] is relatively old (its first flight in 1984). Nevertheless, it has been progressively upgraded with modern technology. The TC-II model includes the Pratt & Whitney Canada PT6A-25C engine, Martin-Baker Mk.11 ejection seats, winglets, and modern digital avionics with synthetic cockpit, advance glass cockpit with Mission Computer, multifunction Displays, and HUD. Therefore, it is a mature technology aircraft that could satisfy the required teaching characteristics.

- **Alternative A4. KTI – Basic Trainer.** This recent aircraft has received a very positive market response, with already 92 operational units worldwide. Its power plant is the Pratt & Whitney Canada PT6A-62 turboprop engine, which exhibits lower power than the other candidates. The other characteristics are similar: zero–zero ejection seat, cockpit pressurization, On-Board Oxygen Generating System etc. [28]. It is also equipped with HUD, UFCP, multifunction displays (MFD), and an open architecture mission computer [29].

- **Alternative A5. CASA C-101 Aviojet.** The C-101 aircraft is also included as an alternative for comparison with the new proposed models. Between 2009 and 2011, several SPAFA test pilots and flight instructors performed flight tests with the different proposed aircrafts in order to compare them with the CASA C-101 Aviojet. These tests could be used to evaluate whether a full and exhaustive modernization of the current trainer would be justified. Carrying out a modernization program would solve the corrosion and fatigue problems as well as the replacement of the avionics, cockpit, and security systems.

3.3. Criteria selection

Regarding the quantitative criteria, only the most important technical parameters for the aeronautical training were considered [1,14,32,33,46]. The qualitative criteria were supported by the test pilots’ impressions during the flight tests. It is important to highlight that this work is a preliminary study that can be used to guide a more in-depth analysis that should take into account relevant aspects such as commercial strategies between countries, counterparties, and economical issues (acquisition and maintenance costs), as well as technical details, such as weapon system sustainability along the service life of the aircraft.

The criteria used were the following:

- **C1: Service ceiling (ft),** the altitude at which the maximum achievable rate of climb is 100 ft/min. According to the SPAF-EMA [42] the minimum service ceiling is 25 000 ft, with a desired value of 30 000 ft. This value should be maximized.

- **C2: Cruising speed (kt),** the constant and uniform speed at which an aircraft is able to fly at a given altitude and for a given power (normal cruise power). This criterion should also be maximized. Its value must be greater than 265 kt at 15 000 ft. [42].

- **C3: Stalling speed (kt),** the lowest speed at which an airplane can fly in straight and level flight. This value should be minimized.

- **C4: Endurance (minutes),** the maximum time that an airplane stays in the air on a tank of fuel. This value should be maximized. Endurance must be greater than 150 minutes [42].

- **C5: Positive Limit Load Factor (+G),** the maximum value of positive load factor which the airframe can withstand without structural damage. This criterion should also be maximized. As a minimum, a value of +7 G is considered, with a desirable value of +8 G [42].

- **C6: Negative Limit Load Factor (−G),** the maximum value of negative load factor which the airframe can withstand without structural damage. As a minimum, a value of −3 G is considered, with a desirable value of −4 G [42].

- **C7: Take-off distance (ft),** the minimum running length along the ground required by an airplane, starting from zero velocity, to gain flight speed and lift from the ground (in Standard Sea Level conditions). A maximum value of 3200 ft is imposed, desiring a minimum value [42].

- **C8: Landing distance (ft),** the minimum distance between the touchdown point and the point where the airplane’s motion stops (in standard sea level conditions). This criterion should also be minimized. The maximum allowed value is 4000 ft [42].

- **C9: Human factors: the comfort conditions inside the cockpit (this criteria encompass aspects such as readability of displays, design of switches and controls, adjustability of seat and pedals, noise levels, etc.)

- **C10: Flying and handling qualities, a flight test criterion used to evaluate aircraft handling qualities based on pilots’ feedback regarding controllability, workload, and the ability to attain adequate performances.

- **C11: Security systems, devices of the aircraft to deal with setbacks or unexpected situations (ejection systems, sensors, etc.).

- **C12: Tactical capability described as the aircraft’s ability to adapt to different training levels and several mission roles such as CAS (Close Air Support), CSAR (Combat Search and Rescue).

It must be noted that criteria C9, C11, and C12 were characterized in qualitative terms by means of linguistic assessment labels.
Criterion $C_{10}$ were objectively scored by the expert test pilots using the Cooper–Harper scale [20].

3.4. Problem structure

Based on previous considerations, the trainer selection can be considered as a MCDM problem [7,25,30], where the best alternative is sought between the $A_i$, $i = 1, 2, \ldots, n$, with $n = 5$ when considering the criteria $C_j$, $j = 1, 2, \ldots, m$, with $m = 12$. A hierarchical structure with two levels will be used to represent this problem (Fig. 2).

3.4.1. Expert survey

The extraction of knowledge from the group of experts is addressed by a survey based on a pseudo-Delphi technique, since the members who are part of the decision do not interact at any time. In order to do this, a survey was distributed among the participants so that they could choose the answers they considered most appropriate. The questionnaire comprised of two distinct parts:

1. The first one presented the decision problem detailing the variables used and the work methods to be carried out. This helped to obtain a hierarchical structure which includes the fundamental elements of the decision problem to enable the choice of the best alternative. The experts were asked if all the criteria were equally important from their point of view. All three experts gave a negative answer and, thus, it was proposed to carry on with the second part of the survey.
2. The second part of the survey was based on the hierarchical structure proposed with the aim to collect data.

3.4.2. Determination of the weights of the criteria

To determine the weights of the criteria, a process analogous to the one explained in [17] was performed. A 3 question survey was designed to apply the AHP methodology in order to obtain the weights of the criteria by each of the experts. Subsequently, considering that all experts were equally important in the decision problem, a homogeneous aggregation of the different experts’ results was performed by arithmetic average. The obtained results (fuzzy numbers) for the weights of the criteria are shown in Table 3.

The results showed that the most important criteria are $C_{11}$ (security systems), $C_{10}$ (flying and handling qualities), and $C_9$ (human factors), while the least important criteria are $C_5$ and $C_6$ (positive and negative limit load factors).

In order to ensure the consistency of the AHP method, the consistency ratio (CR) for each expert was calculated, resulting in a value lower than 0.1 for all cases. Consequently, according to [40], the consistency of the method is ensured.

3.4.3. Obtaining the assessments of the alternatives

Once the weights of the criteria were obtained, the alternatives for each of them were evaluated. In this step, criteria $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, $C_6$, $C_7$, and $C_8$ were numerically quantified from literature data [4,37,38,43,46,49] and the trained test pilots’ reports; the obtained values for these criteria are shown in Table 4. Criteria $C_9$, $C_{11}$, and $C_{12}$ were evaluated through linguistic labels, and criterion $C_{10}$ was objectively scored using the Cooper–Harper scale.

The survey given to the SPAFA trained pilots was focused on the assessment of the alternatives for the four last criteria. This evaluation consisted of a four question poll. In order to make these questions, two kinds of linguistic labels, $L_1$ and $L_2$, were used for criteria $C_9$, $C_{11}$, and $C_{12}$. The linguistic labels used to describe the

![Hierarchy structure of the problem.](image)

**Fig. 2.** Hierarchy structure of the problem.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Weights of criteria through experts’ homogeneous aggregation.</th>
</tr>
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<tbody>
<tr>
<td>Experts’ homogeneous aggregation</td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>0.033, 0.044, 0.058</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.037, 0.051, 0.069</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.039, 0.055, 0.077</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.033, 0.044, 0.059</td>
</tr>
<tr>
<td>$C_5$</td>
<td>0.032, 0.042, 0.053</td>
</tr>
<tr>
<td>$C_6$</td>
<td>0.030, 0.038, 0.049</td>
</tr>
<tr>
<td>$C_7$</td>
<td>0.042, 0.061, 0.091</td>
</tr>
<tr>
<td>$C_8$</td>
<td>0.038, 0.053, 0.074</td>
</tr>
<tr>
<td>$C_9$</td>
<td>0.081, 0.113, 0.165</td>
</tr>
<tr>
<td>$C_{10}$</td>
<td>0.108, 0.137, 0.175</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>0.250, 0.294, 0.326</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>0.047, 0.070, 0.107</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Table 4</th>
<th>Quantitative assessments of criteria $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, $C_6$, $C_7$, and $C_8$ [4,37,38,43,46,49].</th>
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<tr>
<td>$C_1$ (ft)</td>
<td>$C_2$ (kt)</td>
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<tr>
<td>$A_1$ 38000</td>
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<tr>
<td>$A_2$ 31000</td>
<td>316</td>
</tr>
<tr>
<td>$A_3$ 32152</td>
<td>283</td>
</tr>
<tr>
<td>$A_4$ 30000</td>
<td>310</td>
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<tr>
<td>$A_5$ 42000</td>
<td>354</td>
</tr>
</tbody>
</table>
alternatives for each of the criteria, their quantitative assessment, and their fuzzy numbers are shown in Table 5.

Regarding criterion C9, the different trainers were assessed using type L1 linguistic labels. For criteria C11 and C12, different alternatives were assessed through type L2 linguistic labels. These criteria should be maximized.

Finally, with respect to criterion C10, the alternatives were assessed using the Cooper–Harper rating scale, which is summarized in Fig. 3 (from [20]). The pilot rating in this scale should be minimized.

In Fig. 4 a sketch of the decision making matrix is shown in order to summarize all the criteria assessments provided by the experts and the rest of the criteria for each of the alternatives. In this matrix, rows are related to the different alternatives (Section 3.2) and columns are related to the criteria (Section 3.3 and 3.4).

Once the set of criteria has been selected, and the weights of the criteria obtained as explained in Section 3.4.2 (Table 3), a measure of the effect produced by each alternative with respect to each criterion is needed. To do this, the TOPSIS method was used.

The TOPSIS method is particularly useful for problems in which the valuations of the alternatives on the basis of the criteria are not represented in the same units [17]. This is the case in this paper, since there are numerical values for criteria C1, C2, C3, C4, C5, C6, C7, C8, and C10 and linguistic labels for criteria C9, C11, and C12.

In this problem, by considering both numerical values and linguistic labels modelled by triangular fuzzy numbers, the TOPSIS method was adapted to fuzzy set theory, including the defuzzification process [6, 10, 11, 17].

3.5. Results

The fuzzy TOPSIS method provides a ranking of the alternatives with fuzzy numbers according to their relative proximity. This fact is sketched in Fig. 5. By means of the defuzzification process, which transforms the fuzzy numbers into crisp values [17], a ranking (R) of the alternatives with real numbers can be obtained.

![Fig. 3. The Cooper–Harper rating scale [20].](image_url)

![Fig. 4. The decision-making matrix.](image_url)

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**Table 5**

Linguistic label type.

<table>
<thead>
<tr>
<th>Linguistic label $L_1$</th>
<th>Linguistic label $L_2$</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very bad VB</td>
<td>Very low VL</td>
<td>[0.1, 3]</td>
</tr>
<tr>
<td>Bad B</td>
<td>Low L</td>
<td>[1.3, 5]</td>
</tr>
<tr>
<td>Medium m</td>
<td>Medium m</td>
<td>[3.5, 7]</td>
</tr>
<tr>
<td>Good G</td>
<td>High H</td>
<td>[5.7, 9]</td>
</tr>
<tr>
<td>Very good VG</td>
<td>Very high VH</td>
<td>[7.9, 10]</td>
</tr>
</tbody>
</table>

![Sketch of the decision-making matrix](image_url)
The results for the studied problem are shown in Table 6. For the sake of brevity, the theoretical details have been omitted. The interested reader can derive them from the procedure described in the algorithm in Fig. 1 and [17].

From the results in Table 6, it can be concluded that the best alternative, with a crisp value after the defuzzification process of 2.4, is the A1 (Pilatus PC-21). The obtained values for the second and third options are similar, where the second best alternative is the A2 (Beechcraft T-6C) and the third is the A3 (CASA C-101 Aviojet). It is noteworthy that the two last alternatives (A3 and A4) obtain values significantly lower than the rest.

The obtained results were shown to the trained test pilots, who confirmed that the obtained result was in line with their subjective expectations. Additionally, modern systems and equipment could improve the student training for advanced specialized courses, without increased operative effort.

According to the experts, the second position of the Beechcraft T-6C is justified because there are certain details that make it worse than the Pilatus option. For example, while the student position is quite comfortable, the instructor position is rather uncomfortable with reduced front visibility. Additionally, the aircraft exhibits some yaw effect which makes directional control more difficult. Finally, the central display shows too much data potentially causing confusion.

The third alternative (A5) corresponds to the aircraft currently used by the SPAFA (CASA C-101 Aviojet). This result indicates that a deep economic and viability study should be performed in order to determine whether a complete modernization program is justified. This program should consider the corrosion and fatigue airframe problems, as well as the modernization of the avionics, security systems and cockpit.

### 4. Conclusions

In this work, the MCDM method was applied to determine the best trainer aircraft among a set of alternatives proposed by the SPAFA EMA and MALOG. Quantitative technical data to evaluate the different alternatives was extracted from manufacturer information and flight tests. Additionally, insight from three senior test pilots and flight instructors who piloted the considered aircraft, was obtained in the form of linguistic labels via surveys. This information was modelled using triangular fuzzy sets. With this data, a formulation of the TOPSIS method for fuzzy numbers was applied, after using the AHP methodology in order to obtain the weight of the criteria.

As a result of the process, the Pilatus PC-21 aircraft of the Swiss company Pilatus Aircraft Ltd. was deemed the best alternative. The trained test pilots confirmed that the obtained result was in line with their subjective expectations. It is important to note that the current SPAFA jet trainer (CASA C-101 Aviojet) obtained a fairly high ranking, very close to the second best option, the American Beechcraft T-6C aircraft.

Finally, it should be emphasized that this work is a preliminary study taking into account significant technical criteria and the experience of an advisory group composed of trained test pilots and flight instructors of the SPAF. In order to further extend this work, a study should be carried out, taking into account additional important factors, such as business strategies across countries, economic aspects (acquisition and maintenance costs), and weapon system sustainability throughout its operational life.

### Conflict of interest statement

None declared.

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### References


