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Robust analysis for downside risk in portfolio management for a volatile stock market

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ABSTRACT

Variance and downside risk are different proxies of risk in portfolio management. This study tests mean–variance and downside risk frameworks in relation to portfolio management. The sample is a highly volatile market; Karachi Stock Exchange, Pakistan. Factors affecting portfolio optimization like appropriate portfolio size, portfolio sorting procedure, butterfly effect on the choice of appropriate algorithms and endogeneity problem are discussed and solutions to them are incorporated to make the study robust. Results show that downside risk framework performs better than Markowitz mean–variance framework. Moreover, this difference is significant when the asset returns are more skewed. Results suggest the use of downside risk in place of variance as a measure of risk for investment decisions.

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1. Introduction

Risk-return relationship is long considered as the backbone in portfolio management (Elton et al., 2003). Firms hedge and construct portfolios to guard against financial risk as a part of risk mitigation strategy. Measures of risk used, however, are largely debated in the extant literature[3] with no consensus on the choice of risk measure. Traditionally, in a mean and variance (MV) framework, the latter has been used as a proxy for risk which, in turn, assumes that the investor gives equal weights to both upside and downside risks (Markowitz, 1952, 1959). The downside risk (DR) framework, on the other hand, is largely based on the concern of investors for safety from a disaster rate (Estrada, 2002; Post and Levy, 2005; Roy, 1952).

Motivation for testing the two frameworks in a volatile market is driven by the fact that appropriate measures of risk become crucial to individual and organizations in markets that are marked by high uncertainty. During volatile times, many investors are concerned and question their investment strategies in terms of asset allocation. Moreover, robust studies require addressing different issues[4] related to portfolio optimization for both frameworks which previous studies lacked. Though solutions are proposed in different literatures but a comprehensive study for investors, institutional investors and researchers alike for financial modeling is needed. Primarily, asset allocation ensures value for the stakeholders in financial markets. This study will help in anticipating portfolio risk for the investor, explain the market behavior, the nature of investment and can be pivotal for an institutional demand of common stocks.

Harlow (1991) and Foo and Eng (2000) compare MV and DR frameworks but they ignore the effect of skewness and high correlation between variance and downside risk. Our contribution to the existing literature is that we study the alternative models based on sorted portfolios on skewness and measures of risk; variance and downside risk which is previously neglected. Secondly, we address a number of key issues like appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem. These issues have not been discussed in one study and are normally considered as contributing to the contrasting results in the empirical studies.

Thirdly, we contribute to the relevant literature by providing empirical evidence on both MV and DR frameworks in a volatile emerging market such as Pakistan. Haque et al. (2004) comment that the safety-first rule offers minimization of the chance of large negative returns. This is appropriate for emerging markets as their equity distributions are subject to extreme returns. Lastly, we use an index to assess differences between MV and DR frameworks. This index will help in assessing the magnitude of difference in portfolio composition under alternative models.

The results of this study report that DR framework is more efficient than MV framework especially when skewness is high. On the other

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[3] See e.g., Markowitz (1952, 1959), Estrada (2002) and Post and Levy (2005).
[4] i.e. appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem.

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hand, the former is relatively less efficient compared to the latter when skewness is on the low side. These results are based on different portfolio sorting methods. It can be safely concluded that downside risk, as a proxy of risk, is a better measure compared to the variance. Moreover, it is reported that difference in portfolio composition under alternative models is also substantial that cannot be ignored.

The article is organized as follows; Section 1 is the introduction and the next section starts with a brief description of proxies of risk; variance and downside risk. This is followed by discussions on factors affecting portfolio optimization. Section 2 covers data and methodology while Section 3 presents the results. Last section is conclusion and recommendations.

1.1. Variance and downside risk as a measures of risk

Markowitz (1952) assumes that the investment decision is made on the parameters of return and risk. Stock returns are assumed jointly normal to justify the variance as a proxy for risk (Hwang and Pedersen, 2002). The criterion to be jointly normal is that the stock returns have to be individually normal as well while the converse is not true (Shanken, 1982; Zhou, 1993). Contrary to the condition of normality assumed, stock returns are found to exhibit skewness and kurtosis.[5] These findings make variance as the proxy of risk questionable, especially under large departure from normality and when the distribution is severely asymmetric (Athayde and Flôres, 2004; Chunhachinda et al., 1997; Harvey et al., 2010; Jondeau and Rockinger, 2006).

Similarly, Roy (1952) argues that the investor care for disaster following safety-first rule. Moreover, there is evidence of investors assigning different weights to upside and downside risk (Estrada, 2002, 2007; Gul, 1991; Kahneman and Tversky, 1979; Post and Levy, 2005). As investors prefer the safety from disaster and, furthermore, stock returns do not depict the normal distribution, DR measure is a better choice over variance as a proxy for risk (Atwood et al., 1988; Foo and Eng, 2000; Harlow, 1991; Sing and Ong, 2000; Swisher and Kasten, 2005).[6] Cheng and Wolverton (2001) compare MV and DR frameworks and conclude that the latter is a viable alternative compared to the former. Brogan and Stidham (2005) report that DR is consistent with the way investors perceive risk. Moreover, Schindler (2009) studies the co-movements between various asset returns and questions the application of MV optimization. Kroencke and Schindler (2010), Kuzmina (2011) and Sévi (2013) state the practical viability of DR framework in portfolio optimization.

1.2. Optimization model

Insight to the comparative analysis of the two alternative risk measures; variance and downside risk, leads to the construction of efficient frontiers using convex optimization. Markowitz (1952, 1959) primarily asserts that variance is the only measure of risk. He also contests that all, apart from systematic risk, can be diversified away. In this case, the variance of a portfolio is weighted covariance between individual securities as:

$$\sigma^2_{MV-P} = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} W_i W_j \sigma_{ij}. \tag{1}$$

Subject to the following constraints:

$$C1 : \sum_{i=1}^{n} W_i = 1$$
$$C2 : W_i > 1 \text{ for short sales not allowed}$$

where σ^2_{MV-P} is the variance of the portfolio based on MV framework, W_i and W_j are weights of individual securities i and j and σ_{ij} is the covariance between securities i and j. $C1$ implies that all weights should be equal to 1. $C2$ is short sales not allowed indicating all weights are non-negative.

Using Eq. (1), Markowitz's (1952) efficient frontier can be constructed assuming efficient diversification and without unlimited borrowing and lending following Harlow (1991), Foo and Eng (2000), Boasson et al. (2011) and Rasiah (2012). DR framework is incorporated in Eq. (1) by replacing variance by proxy for downside risk as Asymmetric Lower Partial Moments (ALPM)[7] subject to the same constraints as in Eq. (1) as follows:

$$\sigma^2_{DR-P} = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} W_i W_j ALPM_{ij} \tag{2}$$

where σ^2_{DR-P} is the variance of the portfolio based on DR framework and $ALPM_{ij}$ is the covariance between securities i and j such that the covariance between securities i and j is not necessarily equal to j and i.[8] Researchers like Harlow (1991), Foo and Eng (2000), Boasson et al. (2011) and Rasiah (2012) construct efficient frontiers for both the alternative models using Eqs. (1) and (2). Convex optimization is used and comparison between variance and downside risk as a measure of risk is investigated. However, there are factors that cannot be ignored that influence the optimization process, and in return, affecting the efficient frontiers. They are discussed in the following section.

1.3. Factors affecting portfolio optimization

Portfolio optimization is the process which identifies the appropriate proportions of various assets to be held in a portfolio. The criterion for it is based on portfolio returns and the dispersion of returns along with the covariances involved in the portfolio optimization process. Different factors like appropriate portfolio size, sorting procedures, butterfly effect on the choice of appropriate algorithms and the endogeneity problem affect this process. These factors are important to make the process as well as the study robust. This study discusses these factors comprehensively and applies them in MV and DR frameworks to yield robust results.

1.3.1. Portfolio size determination

Inappropriate allocations, the inclusion of inappropriate assets and non-uniqueness of the optimizer solutions are common problems in portfolio optimization arising due to the absence of an appropriate portfolio size. The nexus of feasible and appropriate portfolio size is directly linked to the benefits of diversification. Evans and Archer (1968) propose that around 10 stocks are benchmarked to attain the benefits of diversification. Similarly, Elton and Gruber (1977) use equal-weighted portfolios and report that 10–15 stocks in a portfolio is an appropriate figure. Statman (1987) concludes that 30–40 stocks are sufficient for a well-diversified portfolio. Fama and French (1992) use 25-stock portfolio for their study while Byrne and Lee (2000) advocate the use of 20–40 stocks in a portfolio for naïve investors to make a well diversifiable portfolio.

[5] Evidence for skewness see Eftekhari and Satchell (1996); Bekaert and Harvey (1997), Hwang and Pedersen (2002), Dufour et al. (2003), Sheikh and Qiao (2010) and Ramos et al. (2011).

[6] For a comprehensive literature review on both risk measures, see Abbas et al. (2011).

[7] Bawa (1975), Fishburn (1977) and Bawa and Lindenberg (1977) are the major three who contributed in specifying the proxy for downside risk.

[8] Unlike Markowitz (1952) where covariance of security i and j is equal to covariance of j and i.

Campbell et al. (2001) disagree to their predecessors, and contest that 50 randomly picked stocks contribute to full diversification. Tang and Tsitsiashvili (2004) suggest that 20 stocks in a portfolio eliminate 95% of diversifiable risk while Zulkifli et al. (2010) report that a well diversifiable portfolio consists of 15 stocks. Frahm and Wiechers (2011) report that 60% of total portfolio risk is diversified using 40 stock portfolio. Recently, Alekneviciene et al. (2012) propose 22-stock portfolio and Chong and Phillips (2013) advocate that 20–30 stock portfolio is sufficient for efficient diversification. With much diversity in recommended portfolio size ranging from 10 to 50 stocks, portfolio optimization process becomes increasingly difficult. An appropriate portfolio size is important to avoid inappropriate allocations, the inclusion of inappropriate assets and non-uniqueness of optimizer solutions.

1.3.2. Portfolio sorting

Another factor affecting the portfolio optimization is a portfolio sorting procedure. Portfolio sorting is not new in finance and it can take the form from single to higher order sorting. The single sorting procedure is used by Jegadeesh and Titman (1993, 2001), Korajczyk and Sadka (2004) and Sadka (2006). Double-sorted portfolios are formed by De Bondt and Thaler (1987), Fama and French (1992, 2004), Lakonishok et al. (1994), and Ang et al. (2006) while triple-sorting portfolios on various factors are formed by Daniel and Titman (1997) and Vassalou and Xing (2004).

Portfolio sorting is considered a priority process of optimization in which effects of factor loadings are disentangled. It is important to control the effect of a factor and disentangle the effect of high correlation among different factor loadings. There are three primary factor loadings related to this study; skewness, downside risk and variance, that can lead to biased portfolio optimization. Skewness is an important criterion for sorting as stock returns depict non-normal distribution.[9] It is necessary to sort the portfolios on skewness to explicitly control the effect of skewness in the stock returns. Moreover, another factor loadings for portfolio sorting are variances and downside risk (Ang et al., 2006). They have high correlation among themselves necessitating that this effect has to be disentangled (Post and van Vliet, 2004).

1.3.3. Portfolio optimization algorithm and butterfly effect

Appropriate selection of algorithm is necessary to build efficient frontiers for convex optimization. An inappropriate algorithm can be sensitive, limited, complex, unstable and may be sensitive to the butterfly effect[10] causing disparity for efficient frontiers (Nawrocki, 2000). There are four major types of algorithms that can be used for convex optimization to build efficient frontier. The first three among them are Martin's (1955) calculus minimization—simultaneous equations optimizing algorithms, general nonlinear programming optimizing algorithms and Frank and Wolfe (1956) algorithms. Minimization—simultaneous equations optimizing algorithms and general nonlinear programming optimizing algorithms are both sensitive to butterfly effect.

Frank and Wolfe (1956) algorithm eventually slows down substantially when it convex to minimum point. In case of sub-linear convergence, it performs worst and often is observed to yield an approximate solution. The fourth algorithm is developed by Markowitz (1959) as critical line optimizing algorithm which is a better choice as compared to the other three algorithms. It is considered best algorithm for adding constraints to the optimizer and is least sensitive to the butterfly effect (Nawrocki, 1996, 2000). Markowitz (1987) strongly recommends critical line optimizing algorithm as it is robust to rank matrix errors, thus has an ability to handle large number of stocks.

[9] Evidence for skewness see Ramos et al. (2011).

[10] Butterfly effect, important in chaos theory, is defined as small change in input works through the system to yield large resultant changes.

1.3.4. Endogeneity problem and semivariance optimization

Asymmetry of semivariance algorithms in Eq. (2) for downside risk can yield endogenous cosemivariance matrix which is not positive semi-definite matrix. This leads to the absence of a closed-form solution which is a necessary condition for optimal. When portfolio underperforms benchmark, Eq. (2) requires a laborious iterative process as cosemivariance matrix is endogenous. Indignity is ensured as a change in weights subsequently affecting the elements of the semicovariance matrix via its periods (Cumova and Nawrocki, 2011; Estrada, 2008). Researchers have provided solutions to make endogenous cosemivariance matrix exogenous. Hogan and Warren (1972) use a two-step approach to solve the problem of endogeneity in cosemivariance matrix. Ang (1975) applies programming to yield semicovariance matrix and Nawrocki (1991) suggests a constant correlation heuristic.

Similarly, Harlow (1991) and Grootveld and Hallerbach (1999) build an efficient frontier from semicovariance matrix. However, most of these findings are rejected as some require very high computation resources while some assume unrealistic assumptions like assuming pairwise correlation are the same. Moreover, some apply linear programming instead of quadratic whereas in some cases, below mean semicovariance is inconsistent with utility theory. And even fewer do not specify their methodologies. Estrada (2008) proposes that endogenous cosemivariance matrix is constricted on all elements of the covariance matrix. More recently, Cumova and Nawrocki (2011) also offer an appropriate solution by taking the mean of endogenous cosemivariance matrix and its transpose.

2. Data and methodology

This study chooses a sample of firms listed on Karachi Stock Exchange (KSE), Pakistan. KSE is characterized by high turnover and high price volatility, and moreover, it is an important emerging market. Due to strong institutions like KSE, Pakistan's economy has been included in four emerging markets list by Dow Jones. This list represents those economies that are expected to contribute to global growth.[11] Despite the small size of the market, it experienced high turnover of 323% as compared to NYSE having turnover 65% in 2006. KSE-100 rose by 650% from 2001 to 2005 while Bombay (Mumbai) Stock Exchange (BSE30) experienced a rise of 137% for the same period making KSE an important emerging market.[12]

KSE was declared as Best Performing Stock Market of the World by Business Week in 2002. KSE-100 experienced an unprecedented rise of 65%, from 6218 on December 31, 2004 to 10,303 on March 15, 2005. Henceforward, the market turned negative and index declined by 32.7% from its peak. In April 2008, KSE crossed 15,000 barrier first time in its history. However, in May 2008, SBP increased interest rates unexpectedly to address high inflation, consequently, KSE declined sharply with a one-third drop in its index and in August, 2008 the floor was halted. Afterwards, trading was allowed to resume on December 15, 2008 and amazingly, KSE recovered 20–25% of its decline in the index till March 12, 2009. In December 2010, index steadied around 12,000 points showing unparalleled attitude towards volatility and turnover.

This study randomly selects data for 100 listed-companies of KSE, Pakistan from 2000 to 2010. However, as trading was halted from August, 2008 to December, 2008, this period has not been included in the sample. Following Javid and Ahmad (2008),[13] this study selects

[11] www.dowjones.com.

[12] www.world-exchanges.org.

[13] This may produce survivorship bias. However, there are no formal databases present in Pakistan in which complete history of deleted companies is present. Additionally, recent studies on KSE-Pakistan has not addressed this issue due to non-availability of data (see e.g., Ahmad and Qasim, 2004; Iqbal and Brooks, 2007; Javid and Ahmad, 2008; Iqbal et al., 2010).

only those companies which are listed throughout the sample period.[14] These companies are randomly selected from almost all types of sectors. This study forms one-year-investment-horizon portfolios based on 5-year of monthly ex post data following Black et al. (1972), Fama and MacBeth (1973), Fama and French (1992) and Ang et al. (2006). After every one year, portfolios are again reconstructed using 5 year historical data to make it rolling window analysis.

This study tests MV and DR frameworks by constructing efficient frontier and then comparing between alternative frameworks. To make the study robust, factors affecting portfolio optimization are addressed and feasible solutions are incorporated. Major concerns are appropriate portfolio size, portfolio sorting procedures, butterfly effect on the choice of appropriate algorithms and endogeneity problem which the study covers. Lastly, the issue of the significance of difference between alternative frameworks is addressed. The study adopts numerous remedies for the factors affecting portfolio optimization which are discussed henceforth.

2.1. Determining appropriate portfolio size

This study primarily follows Evans and Archer (1968) and Chong and Phillips (2013) to determine the appropriate portfolio size. However, unlike Evans and Archer (1968), this study ignores dividends as stock prices are automatically adjusted for them for a stock listed at KSE, Pakistan. Moreover, the arithmetic mean is used instead of geometric mean for each stock. This study uses equal-weighted portfolios following Evans and Archer (1968), Fama and MacBeth (1973), Harvey and Siddique (2000), Korajczyk and Sadka (2004) and Ang et al. (2006) to make results more robust.[15] This study constructs portfolios with the number of stocks ranging from 2 to 60 in each portfolio. After 60, blocks of portfolios with 70, 80, 90 and 100 stocks are constructed in each of them. For each portfolio group, 7 observations are taken of standard deviation (STD) from 2004 to 2010 and then average STD is taken.

In all, 640 portfolios are constructed from companies listed at KSE-Pakistan. This study does not perform regression analysis as employed by Evans and Archer (1968) and Chong and Phillips (2013), rather, this study follows Elton and Gruber (1977) and Brown et al. (2012) using ratio scale. Ratio scale helps to assess drop of risk due to portfolio size.[16] However, this study does not calculate average STD for a base of 1-stock portfolio of sample companies as done by Elton and Gruber (1977) and Brown et al. (2012). A base of 1-stock creates extreme values biasness, instead, this study takes base of 2-stock portfolio. Secondly, to assess the benefits of diversification, the approach of this research is more appropriate to compare the benefits of additional stocks in a portfolio to a base of 2-stock portfolio as to 1-stock. Lastly, a minimum number of stocks in a portfolio are two and bases of 2-stock portfolio is the minimum value that can be assigned to qualify it as a base.

2.2. Portfolio sorting procedure

This study prefers triple-sorting portfolio procedure on criteria of skewness, [17] variance and DR. As companies listed at KSE-Pakistan depict the absence of normality so this study introduces skewness in portfolio sorting. Firstly, this study sorts stocks on skewness to sort out its effect into two identical sets of stocks. Then each set is sorted again on the basis of alternative frameworks which are further used for portfolio optimization. Variance based sorted stocks are used to construct DR based efficient frontier while DR based sorted stocks are used to construct variance based efficient frontier. This approach has two major advantages; firstly, it specifies sorting criteria; secondly, it disentangles the effects of skewness and the high correlation between variance and downside risk.

Stocks are sorted on the basis of skewness and the first eighty stocks[18] are chosen which are then again split into two portfolios of high skewness and low skewness. Each portfolio of high and low skewness comprises stocks of 40 each. Then this study re-sorts each skewness-based portfolio into two portfolios based on DR resulting into four datasets, each having 20-stock portfolio. These four datasets are further used for MV optimization. Our portfolios for MV optimization are sorted as high skewness–high DR (HH–MV), high skewness–low DR (HL–MV), low skewness–high DR (LH–MV) and low skewness–low DR (LL–MV). Likewise, this study resorts skewness based portfolios into two portfolios based on variance resulting into four datasets, each having 20-stock portfolio. These four datasets are further used for DR optimization. Our portfolios for DR optimization are sorted as high skewness–high variance (HH–ALPM), high skewness–low variance (HL–ALPM), low skewness–high variance (LH–ALPM) and low skewness–low variance (LL–ALPM).

2.3. Portfolio optimization algorithm

Markowitz (1959) develops a critical line optimizing algorithm (CLA) which does not possess shortcomings of general nonlinear programming optimizing algorithms. It is considered as the best algorithm for adding constraints to the optimizer. It can also handle large number of stocks and is least sensitive to the butterfly effect (Nawrocki, 1996, 2000). Moreover, Markowitz (1987), Nawrocki (1996, 2000) and Cumova et al. (2006) strongly recommend CLA as it is robust to rank matrix errors. This study uses CLA for an optimization solution which is defined as a minimizing function for MV framework as follows:

$$\text{MV optimization}: \quad Min\ z = \sigma^2_{MV-P} - \lambda E(R_{MV-P}) \tag{3}$$

where λ is the slope of objective function varied customarily from zero to infinity. σ^2_{MV-P} and $E(R_{MV-P})$ are the portfolio risk and portfolio return respectively for MV framework. For DR framework, CLA optimization solution is as follows:

$$\text{DR optimization}: \quad Min\ z = \sigma^2_{DR-P} - \lambda E(R_{DR-P}) \tag{4}$$

where σ^2_{DR-P} and $E(R_{DR-P})$ are the portfolio risk and portfolio return respectively for DR framework. Using Eqs. (3) and (4), this study can minimize risk for given return subject to constraints specified in it (Nawrocki, 1996, 2000).

2.4. Assuring positive semi-definite matrix for endogeneity problem

Estrada (2008) provides a simple, accurate, guaranteed solution to assure a positive semi-definite matrix for endogeneity problem. He proposes that endogenous cosemivariance matrix is constricted on all

[14] The induces survivorship bias, however, the material effect of it has yet to be investigated.

[15] Fama (1998) concludes that anomalies seem to decrease and disappear frequently when studies are based on value-weighted portfolios. Secondly, Harvey and Siddique (2000) assert that coskewness is strongest for equal-weighted portfolios. Thirdly, Roll (1981) and Korajczyk and Sadka (2004) conclude that equal-weighted portfolios have higher returns than counter-part. Lastly, the absence of linear relationship has been attributed to smaller stocks (Ang et al., 2006).

[16] Regression analysis does not address this issue comprehensively as the inverse relationship between portfolio risk and portfolio size is taken linearly. However, ratio scale is flexible to accommodate both linear as well as nonlinear relationship.

[17] For details on impact of skewness on asset allocation see Xiong and Idzorek (2011).

[18] Results in Section 3.1 advocate 20-stock portfolio as an appropriate portfolio size for optimization. To achieve this result, first eighty stocks are taken.

elements of the covariance matrix. This study incorporates the solution recommended by Estrada (2008) in Eq. (2) by defining ALPM as:

$$ALPM_{ij,n-1}\left(\tau, R_i, R_j\right) = \frac{1}{t}\sum_{i=1}^{t}\left[\text{Max}\left(0, \left(\tau, R_{it}\right)\right)\right]^{n-1}\left[\text{Max}\left(0, \left(\tau, R_{jt}\right)\right)\right] \tag{5}$$

where n is equal to 2 and τ is the threshold equal to 0% is defined as semivariance Nawrocki (1996, 2000). To ensure positive semi-definite matrix, this study calculates, as a precaution, eigenvalues for each matrix. None of eigenvalues are negative, suggesting that a positive semi-definite matrix will work on the convex maximization problem.[19]

Summing up, this study takes $n = 2$ and $\tau = 0\%$ for severance as a risk measure in DR framework. This study uses triple-sorted portfolios to calculate efficient frontiers for both MV and DR framework. The grand mean of all years from 2004 to 2010 is taken and graphs are plotted for it. HH, HL, LH and LL are four types of sorted-portfolios for comparisons between MV and DR frameworks for each year and for a grand total of all years. However, the question does arise about the significance of difference between these alternative frameworks which is addressed in the next section.

2.5. Significance of difference between MV and DR framework

This study is interested in measuring the significance of difference between MV and DR framework. Normally, once portfolios are selected, its performance against some benchmark is assessed. Some common measures are Sharpe (1966), Treynor (1965) and Jensen (1968) ratios. Conversely, Ang and Jess (1979) report that these ratios exhibit biased relationship with risk measures. This problem is addressed by using Grootveld and Hallerbach (1999) RMSDI[20] to differentiate portfolio composition between MV and DR framework as:

$$RMSDI = \sqrt{1/n\sum_{i=1}^{n}\sum_{j=1}^{n}\left(x_{ij}^{CLPM} - x_{ij}^{MV}\right)^2} \tag{6}$$

where x_{ij}^{CLPM} and x_{ij}^{MV} are investment portions (weights) in the DR and MV frameworks respectively. Stocks i and j represent portfolio class and security respectively, and n represents the number of observations corresponding for each framework. Maximum portfolio size is twenty and there are seven classes of portfolio for each year based on historical data of five years from 2000 to 2010. This study tests DR framework with MV framework using the efficient frontiers. It defines the research question as "*is downside risk a better measure of risk compared to variance in asset allocation?*".

3. Results and analysis[21]

3.1. Results from determining appropriate portfolio size

Table 1 shows that the initial drop of risk is 33% in first five stocks. The risk falls to 49% with ten stocks which is sharp for the first ten stocks, Subsequently, the risk falls slowly with a total portfolio risk amounting to around 12%. For ten to twenty stock-portfolio, the decrease is 8% while from twenty to thirty, risk drops by 4%. Thirty to forty and forty to fifty, risk decrease is 1–2% indicating diversification becoming less supportive. Total risk falls by 64% up to fifty stocks with total portfolio risk of 8.49%. Overall, results propagate portfolio comprising 20 stocks diversify 57% of risk. For 80, 90 and 100 stock portfolios, diversification benefits are insignificant. These results are graphically represented in Fig. 1. On the basis of this study portfolio comprising a maximum of 20 securities is an appropriate choice. However, an optimal solution can be less, but not exceeding, twenty securities.

3.2. Mean–variance vs. downside risk efficient frontiers

This study plots graph using values from both MV and DR optimization solutions for HH, HL, LH and LL portfolios. The MV optimization solution is represented by MV while the DR optimization solution is represented by ALPM. This enables to graphically compare the two alternative frontiers. This study follows Harlow (1991) and takes square roots for both risk measures as standard deviation for MV framework and semideviation for the DR framework to be return-compatible units for plotting purposes. The y-axis represents expected return and the x-axis is labeled as a portfolio standard deviation.

Fig. 2 shows results for high skewness–high DR sorted portfolios for the MV framework (HH–MV) and high skewness–high variance sorted portfolios for the DR framework (HH–ALPM). It clearly shows that the HH–ALPM outperforms HH–MV. The efficient frontier of MV lies inside ALPM efficient set indicating that the latter is more efficient for the same level of expected return/risk. For a given slope, HH–ALPM yields both better expected returns as well as less portfolio risk. As λ increases, this difference tends to decrease, but it is negligible. The largest difference is seen at small values of λ which is more apparent in Table 2. On average HH–ALPM is efficient by 11% as compared to HH–MV for the same level of expected return.

Results from Fig. 3 are for high skewness–low DR sorted portfolios for the MV framework (HL–MV) and high skewness–low variance sorted portfolios for the DR framework (HL–ALPM). It shows that the HL–ALPM is towards the left of HL–MV demonstrating that the former is more efficient for the same level of expected return. In other words, for the same level of return, more risk is diversified away. Table 2 shows that as λ increases, this difference tends to decrease eventually to a minimal value. On average, HL–ALPM is efficient by 4% as compared to HL–MV for the same level of expected return. This value is very low compared to HH-sorted portfolios. Major reason identified is that variance and DR have low values and therefore contribute less to portfolio risk.

Fig. 4 demonstrates results for low skewness–high DR sorted portfolios for the MV framework (LH–MV) and lower skewness–high variance sorted portfolios for the DR framework (LH–ALPM). Again, LH–ALPM opportunity set is efficient as compared to LH–MV. DR framework provides better results for the same level of expected returns than MV framework. Table 2 illustrates that LH–ALPM is approximately 11% more efficient than LH–MV sorted portfolios. HH- and LH-sorted portfolios yield similar results. It shows that the difference between different proxies of risk is more if there is more volatility in returns.

Fig. 5 shows result for low sickness–low DR sorted portfolios for the MV framework (LL–MV) and low skewness–low variance sorted portfolios for the DR framework (LL–ALPM). Again, DR framework based LL–ALPM opportunity set provides better results for the same level of expected returns than MV framework. Table 2 illustrates that LL–ALPM is approximately 4% more efficient than LL–MV portfolios. LL- and HL-sorted portfolios yield similar results. These results are explicit that the difference between two methodologies will be most notable and vice versa for risky portfolios. This conclusion supports the theory that investors care for safety from disaster and are risk-averse for downside risk.[22]

Fig. 6 is explicit for a grand total for portfolios sorted as HH, HL, LH and LL for both MV and DR frameworks. Grand total ALPM distinctly is towards the left of Grand Total MV. The grand total for DR framework provides better results for the same level of expected returns than MV framework. Table 2 demonstrates that the former is more efficient

[19] For details on eigenvalues see Lay (2003).

[20] Root mean squared dispersion index.

[21] Results for Section 3: Figures are in Appendix A and tables in Appendix B.

[22] For details see Roy (1952), Hogan and Warren (1972), Bawa and Lindenberg (1977), Kahneman and Tversky (1979), Harlow and Rao (1989), Gul (1991), Estrada (2000), Estrada (2002, 2007) and Post and Levy (2005).

than the latter by an average of 7% for the same level of expected returns. It can be safely concluded that downside risk as a proxy of risk is a better measure compared to the variance.

The results of this study are similar to Harlow (1991) and Foo and Eng (2000) and amplify that downside risk is a better risk measure compared to variance. Robust analysis yields that the difference in the two alternatives is most pronounced when the skewness is high. Moreover, when the alternative measures of risks have high values; meaning that it represents volatile market, then again the results show significant differences. In this case, investors are vying for DR as a better measure of risk. These facts have not been reported in previous studies according to our knowledge. This conclusion is important in studies which are highly reliant on normal distribution in a highly volatile market.

3.3. Assessing difference between variance and DR using RMSDI

Table 3 shows the results of portfolio composition for risk-averse investor (LPM = 2) in comparison with MV framework. For LPM = 2, an investor has difference in (not of) portfolio composition from MV framework, on average, of 6.39%. This difference is large and cannot be ignored, and given the condition of short sales allowed, it will increase. It signifies that DR framework yields similar results to MV framework under condition of normality and departure from normality results in different outcomes.

This study reports that downside risk is a better measure of risk compared to variance in asset allocation. These results are consistent with the studies of Harlow (1991) and Foo and Eng (2000). However, our results explicitly report that DR framework is largely different from MV framework when skewness is high and/or the difference between DR and MV is more. This leads to the conclusion that, whether the data is normal or not, the former performs better than the latter. However, when skewness is low, the difference between alternative models tends to decrease but cannot be ignored altogether.

4. Conclusion and recommendations

This study helps in assessing different risk measures; variance and downside risk, in asset allocation in a volatile stock market of an emerging economy. This area has been of great interest for different stakeholders. An efficient asset allocation ensures value for the stakeholders in the financial markets. The models based on MV and DR frameworks are tested using the portfolio optimization. Factors affecting portfolio optimization are proactively addressed. Efficient frontiers are solved and plotted under alternative models using tripe sorted portfolios. Moreover, RMSDI is used to assess differences between alternative frameworks. Results show that DR framework outperforms MV framework for all kinds of sorted-portfolios. These results are consistent with Harlow (1991) and Foo and Eng (2000) who show superiority of former over the latter using efficient frontiers thus favoring our research question; "*is downside risk a better measure of risk compared to variance in asset allocation?*".

The results indicate that portfolio comprising 20 stocks diversifies 57% of risk. This leads to optimization solution can be less than, if not more, twenty securities. It is also reported that DR framework is 11% more efficient than MV framework when skewness is high. On the other hand, the former is more efficient than latter by 4% when skewness is on the low side. These results are based on different portfolio sorting methods. Overall, DR framework is more efficient than Markowitz's MV framework by an average of 7% for the same level of expected returns. It can be safely concluded that downside risk, as a proxy of risk, is a better measure compared to the variance. Moreover, difference in portfolio composition under alternative models is, on average, of 6.39%.

Researchers can benefit more if their study is based on financial and non-financial sectors. The former is regulated by International Financial Reporting Standards resulting to different risk paradigm. Similarly, this study did not perform efficiency test for the market portfolio. Therefore, stochastic dominance based tests can be used to determine the efficiency of the market portfolio. The use of conditional and multi-factor models in the downside risk framework is not used in this study. This may enhance the performance of the downside risk framework. If a tracking error is introduced then it can make the model multi-dimensional and more receptive to market forces. Finally, behavioral approach can be incorporated in this field as it is steadily gaining attention of researchers and practitioners alike.

Appendix A

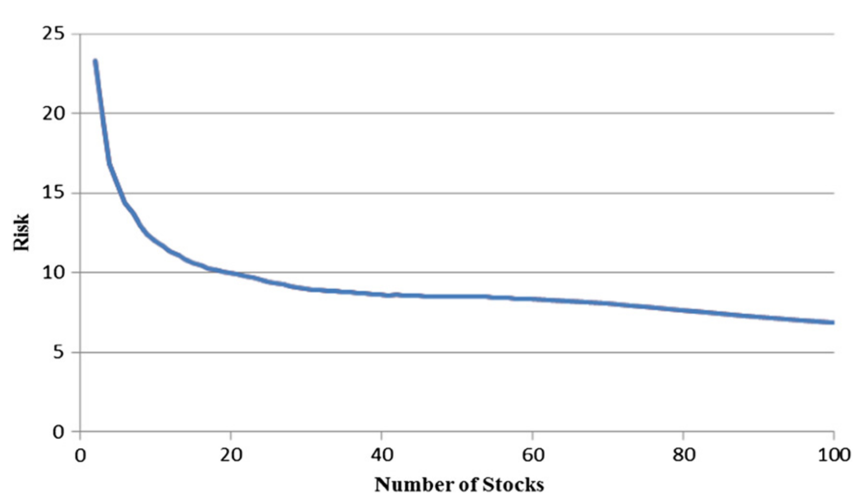

Fig. 1. Benefits of diversification via modified Evan and Archer methodology.

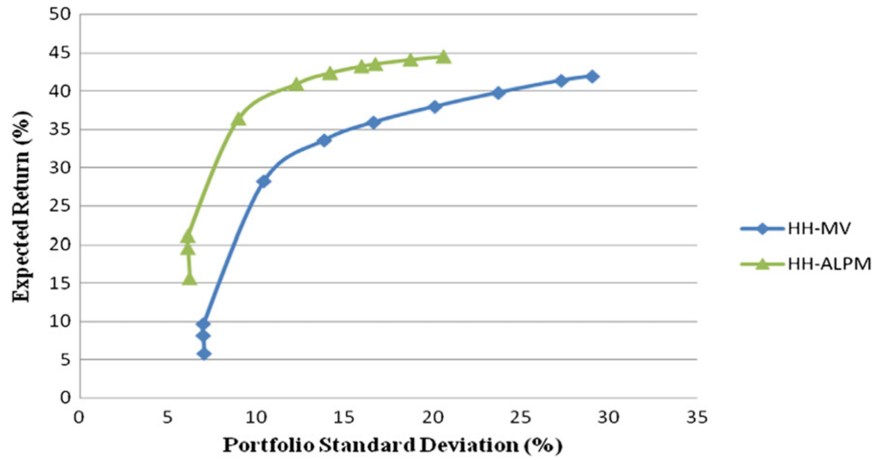

Fig. 2. Results for HH sorted portfolios for MV and ALPM framework.

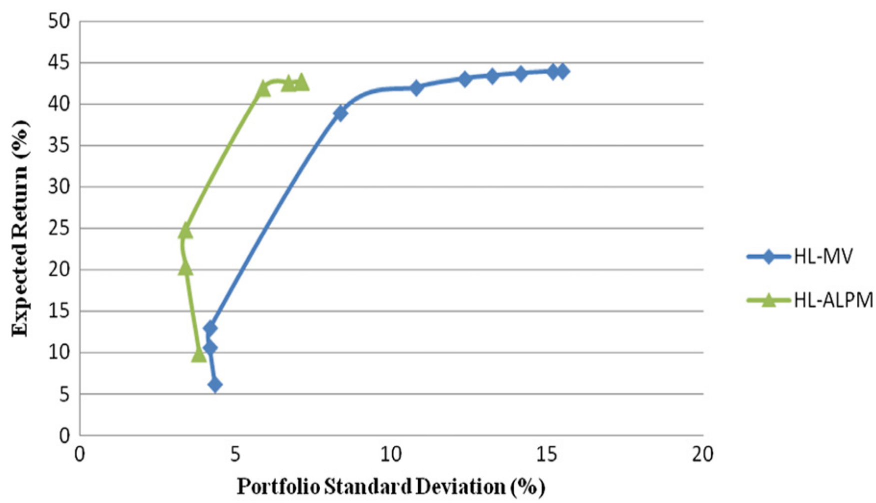

Fig. 3. Results for HL sorted portfolios for MV and ALPM framework.

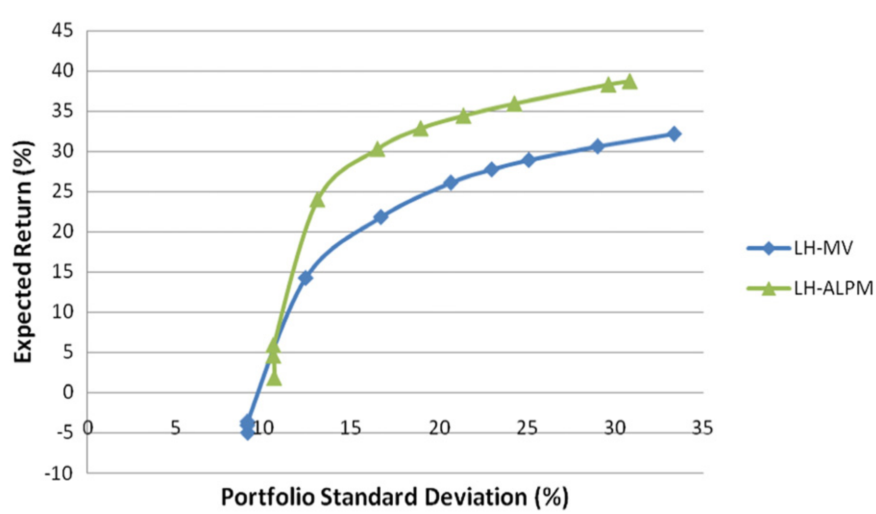

Fig. 4. Results for LH sorted portfolios for MV and ALPM framework.

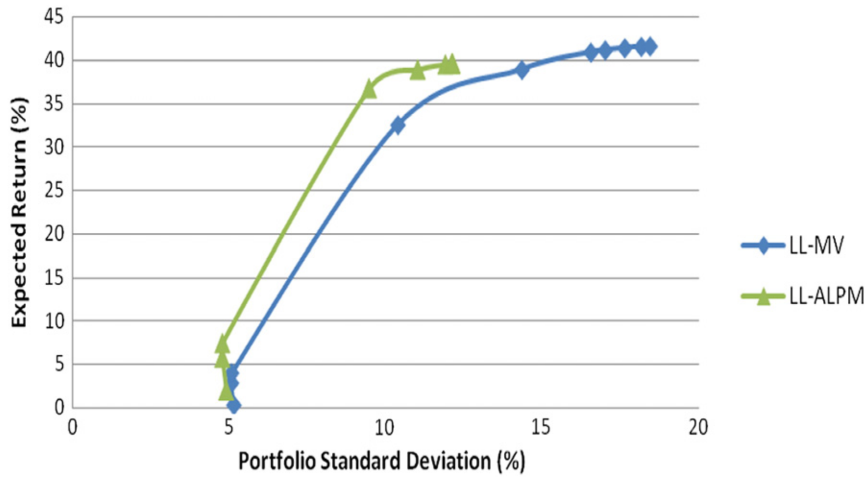

Fig. 5. Results for LL sorted portfolios for MV and ALPM framework.

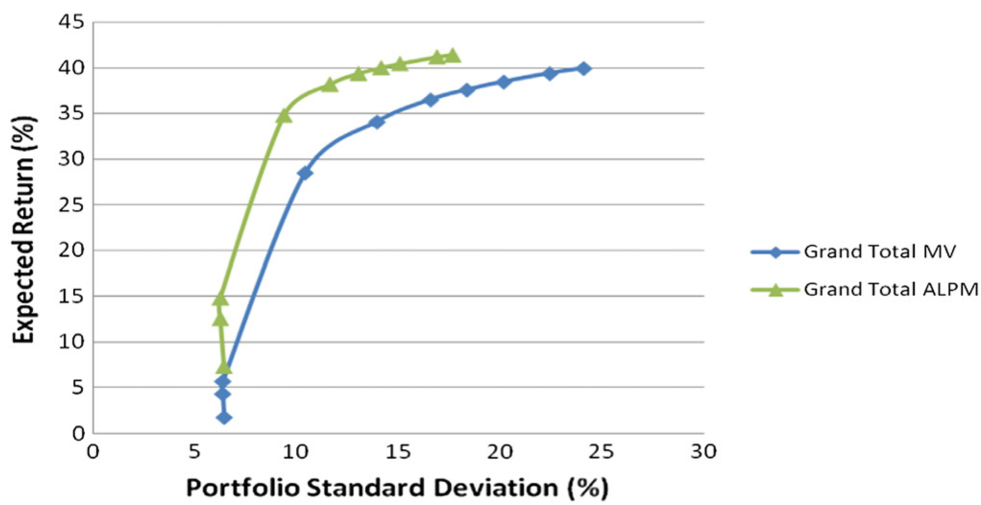

Fig. 6. Results for grand total for all MV and ALPM portfolios.

Appendix B

Table 1
Detail results for modified Evan and Archer. Portfolio risk falls steadily till 3 stocks. From 4 till 10, risk fall is sharp. Till 10-stock portfolio, approximately 50% of risk id diversified.

Number of stocks in portfolio	Evan and Archer STD	Ratio of portfolio STD to 2-stock STD
2	23.33	1
3	19.97	0.86
4	16.84	0.72
5	15.54	0.67
6	14.34	0.61
7	13.71	0.59
8	12.95	0.56
9	12.38	0.53
10	11.94	0.51
11	11.66	0.5
12	11.31	0.48
13	11.09	0.48
14	10.84	0.46
15	10.61	0.45
16	10.44	0.45
17	10.27	0.44
18	10.16	0.44
19	10.04	0.43
20	9.98	0.43
21	9.87	0.42
22	9.77	0.42

(continued on next page)

Table 1 *(continued)*

Number of stocks in portfolio	Evan and Archer STD	Ratio of portfolio STD to 2-stock STD
23	9.67	0.41
24	9.52	0.41
25	9.41	0.4
26	9.31	0.4
27	9.23	0.4
28	9.12	0.39
29	9.07	0.39
30	9.01	0.39
31	8.94	0.38
32	8.89	0.38
33	8.84	0.38
34	8.81	0.38
35	8.77	0.38
36	8.75	0.38
37	8.73	0.37
38	8.69	0.37
39	8.66	0.37
40	8.63	0.37
41	8.59	0.37
42	8.6	0.37
43	8.55	0.37
44	8.57	0.37
45	8.53	0.37
46	8.51	0.36
47	8.5	0.36
48	8.5	0.36
49	8.49	0.36
50	8.49	0.36
51	8.48	0.36
52	8.48	0.36
53	8.47	0.36
54	8.45	0.36
55	8.44	0.36
56	8.42	0.36
57	8.4	0.36
58	8.37	0.36
59	8.35	0.36
60	8.33	0.36
70	8.06	0.35
80	7.64	0.33
90	7.23	0.31
100	6.85	0.29

Column 1 represents the number of stocks in a portfolio. Column 2 represents the standard deviation of portfolio and column 3 represents the risk fall for each portfolio. Results show that the initial drop of risk is 33% for first five stocks. Risk fall falls to 49% till ten stocks which is sharp for first ten stocks, After it, risk-fall slows down with a total portfolio risk amounting to 11.94%. For ten to twenty stock-portfolio, decrease is 8% while from twenty to thirty, risk drop is 4. Thirty to forty and forty to fifty, risk decrease is 1–2%. indicating diversification becoming less supportive. Total risk fall is 64% up to fifty stocks with total portfolio risk of 8.49%.

Table 2
Detail results for different λ for HH, HL, LH and LL portfolio.

	Λ	−0.005	−0.001	0.001	0.1	0.25	0.5	0.75	1	1.5	2
HH	STD–MV	7.06	7.01	7.02	10.44	13.87	16.67	20.14	23.74	27.32	29.07
	ER-MV	5.82	8.17	9.68	28.25	33.59	35.92	37.97	39.78	41.37	41.93
	STD-ALPM	6.24	6.15	6.15	9.01	12.3	14.2	15.99	16.78	18.76	20.63
	ER-ALPM	15.69	19.62	21.2	36.44	40.93	42.37	43.24	43.54	44.1	44.53
HL	STD–MV	4.35	4.19	4.19	8.37	10.8	12.36	13.24	14.15	15.19	15.49
	ER-MV	6.15	10.59	12.95	38.92	41.97	43.04	43.41	43.7	43.94	43.99
	STD-ALPM	3.83	3.4	3.4	5.89	6.71	7.12	7.13	7.13	7.13	7.13
	ER-ALPM	9.86	20.34	24.8	41.92	42.53	42.69	42.69	42.69	42.69	42.69
LH	STD–MV	9.1	9.09	9.09	12.39	16.67	20.64	22.96	25.07	28.98	33.33
	ER-MV	-5.04	-4.11	-3.67	14.21	21.79	26.08	27.71	28.88	30.58	32.14
	STD-ALPM	10.62	10.58	10.57	13.07	16.48	18.93	21.35	24.24	29.57	30.78
	ER-ALPM	1.87	4.6	5.93	24.01	30.27	32.82	34.38	35.89	38.26	38.68
LL	STD-MV	5.15	5.08	5.08	10.4	14.36	16.56	17.03	17.65	18.18	18.46
	ER-MV	0.22	2.76	3.92	32.51	38.89	40.89	41.14	41.38	41.55	41.61
	STD-ALPM	4.92	4.79	4.79	9.48	11.03	11.92	12.13	12.15	12.15	12.15
	ER-ALPM	1.93	5.72	7.39	36.66	38.82	39.41	39.49	39.5	39.5	39.5
Grand total	STD–MV	**6.42**	**6.34**	**6.34**	**10.4**	**13.92**	**16.56**	**18.34**	**20.15**	**22.42**	**24.09**
	ER-MV	**1.79**	**4.35**	**5.72**	**28.47**	**34.06**	**36.48**	**37.56**	**38.44**	**39.36**	**39.92**
	STD-ALPM	**6.4**	**6.23**	**6.23**	**9.36**	**11.63**	**13.04**	**14.15**	**15.07**	**16.9**	**17.67**
	ER-ALPM	**7.34**	**12.57**	**14.83**	**34.76**	**38.14**	**39.32**	**39.95**	**40.4**	**41.14**	**41.35**

HH is portfolios sorted on high skewness–high variance/DR, HL is portfolios sorted on high skewness–low variance/DR, LH is portfolios sorted on low skewness–high variance/DR and LL is portfolios sorted on low skewness–low variance/DR. STD–MV and ER–MV represent standard deviation and expected return respectively for mean–variance framework. STD–ALPM and ER–ALPM represent standard deviation and expected return respectively for Asymmetric Lower Partial Moments framework. λ has ten values from −0.005 to 2. In bottom, grand total is given in boldfaced.

Table 3
RMSDI between portfolio composition between MV and DR frameworks.

RMSDI	MV	LPM = 2
2004	–	7.07
2005	–	6.69
2006	–	6.15
2007	–	8.19
2008	–	5.73
2009	–	8.46
2010	–	2.43
Average RMSDI	–	6.39

Table shows the results of portfolio composition for risk-averse investor (LPM = 2). LPM value of 2 represents semivariance. Year-wise corresponding values are given with average in the bottom row.

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